ST. AUGUSTINE OF CANTERBURY CATHOLIC PRIMARY SCHOOL

Progression in Calculation Policy

Mission Statement

"I called you by your name, you are mine." Isaiah 43

The mission of our school is to support and further the teachings of Christ and His Church.

We welcome and embrace individuals of all abilities and cultural backgrounds. We aim to enhance and celebrate their moral, physical, social and emotional development, so that they may reach their full potential in an atmosphere of stability, care and respect.

We believe that education is for all and in partnership with parents, carers, children and the wider Catholic community: we will strive and succeed in a wholly inclusive setting.

Equality Statement

This policy has been equality impact assessed and we believe that it is in line with the Equality Act 2010 as it is fair, it does not prioritise or disadvantage any pupil and it helps to promote equality at Augustine of Canterbury Catholic Primary School. We have carefully considered and analysed the impact of this policy on equality and the possible implications for pupils with protected characteristics, as part of our commitment to meet the Public Sector Equality Duty (PSED) requirement to have due regard to the need to eliminate discrimination, advance equality of opportunity and foster good relations.

Date adopted: March 2021
Date to be reviewed: June 2022
Written by: Miss Lisa Richardson (Mathematics co-ordinator)

This policy gives an overview of the different strategies used in our school to teach maths throughout the primary maths curriculum. As our children progress at different rates, some may need to use the strategies from previous year groups.

In our school, the children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation.

There is a considerable emphasis on teaching mental calculation strategies. Informal written recording takes place regularly and is an important part of learning and understanding. More formal written methods follow only when the child is able to use a wide range of mental calculation strategies. As children's mental methods are strengthened and refined, so too are their informal written methods. Some recording takes the form of jottings, which are used to support children's thinking. This may be done on scrap paper and whiteboards and is not always retained as it is for the children's own personal use.

This policy contains the key pencil and paper procedures that are taught within our school. It has been written to ensure consistency and progression throughout.

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that our children do not abandon jottings and mental methods once pencil and paper procedures are introduced.

Therefore, our children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose; pictures, mental calculation with or without jottings, structured recording or a calculator.

The national curriculum for mathematics aims to ensure that all children:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that children develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

The overall aims for when children leave primary school are to ensure they:

- have a secure knowledge of number facts
- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10, sums and differences of multiples of 10 and multiplication facts up to 12 x 12
- have a good understanding of the four operations
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally

Points to consider:

Children should not be made to go onto the next stage if:

1) They are not ready i.e. they have not yet secured the pre-requisite skills
2) They are not confident enough yet to move onto the next strategy and need to learn to 'own' it more

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods i.e. always being encouraged to think *'Can I do this in my head?'* first.

Teaching Mathematics at St. Augustine of Canterbury School

Remember:

Every day is a mental mathematics day – ensure that children engage in sustained mental work each day (at least 10 minutes) to secure and develop knowledge, skills and understanding in mathematics. *Don't expect confidence in working mentally if practice, rehearsal and reasoning have not taken place.*

Hands-on learning is still important – provide appropriate practical equipment for children to use and manipulate, to help them to explore how and why things work and to learn to visualise, describe and represent what is in front of them. *Don't just talk about weighing scales, use one; using apparatus is better than imagining how it works.* (***All resources are stored centrally in Year 2 and audited and updated by the Maths Co-Ordinator. Each class has a central maths manipulatives table which is freely available for all children and they are encouraged to use the apparatus (no matter their ability) to explore.)***

Seeing mathematics through models and images supports learning – help children to see how mathematics works and can be represented through physical objects, pictures or diagrams such as place-value cards, number sticks, number lines, representations of fractional parts. *Don't expect children to visualise and 'see' how something works if they have no models and images to draw from.*

Talking mathematics clarifies and refines thinking – give children the vocabulary and language of mathematics; provide activities and time for them to discuss mathematics, using this language. Display the appropriate lesson vocabulary on their interactive whiteboards at the beginning of lessons to ensure the language of mathematics is discussed and referred to throughout. Teach children the precision of language, for example, using: prism, equals, factor and how to express their reasoning using language such as: if... then... , because, cannot be, never, sometimes, always. *Don't expect children to explain or provide reasons if they have no opportunity to use, develop and refine the language to do so.*

Make mathematics interesting – share your interest in mathematics with the children. Give children mathematics that engages them in: estimating and finding out about the number of bricks in the school building, testing out ideas such as when the sum of three consecutive whole numbers is a multiple of six, answering intriguing questions such as how many times their heart beats in ten minutes compared with an elephant or a mouse. *Don't expect children to be interested in mathematics if you don't share an interest and all their mathematics is routine and dull.*

Learning from mistakes should build up children's confidence – look out for mistakes and encourage children to recognise that making mistakes is something everyone does. Show children common errors and get them to identify and correct them. Encourage children to work with a partner and share their work. *Don't just tell children something is wrong; help them to see what went right and to identify when it went wrong.*

DCSF Securing Levels materials, 2009

The following pages show the progression in calculation (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of a concrete, pictorial, abstract approach helps our children develop across all the operations in an efficient and reliable way.

EYFS

Non Negotiables for early number development (based on statements in Development Matters 2020)

Developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently, develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers.

Children in reception will be learning to:

- Count objects, actions and sounds.
- Subitise.
- Link the number symbol (numeral) with its cardinal number value.
- Count beyond ten.
- Count beyond ten.
- Understand the 'one more than/one less than' relationship between consecutive numbers.
- Explore the composition of numbers to 10.
- Automatically recall number bonds for numbers 0–10.

Providing opportunities for daily counting in real-life situations is important, ensuring pupils can:

  - Recite numbers in order and in relation to different sized sets or groups
  - See numerals linked to images of sets, actions and sounds (securing conservation)
  - Rehearse one-to-one correspondence
  - Learn to organise the objects they are counting by placing them in a line

- Daily practice of nursery rhymes and songs to develop memory skills, including using points of transition
- Link number development with stories
- Develop technical language acquisition – ensure that correct mathematical language is used, encouraged, explained and listed on planning
- Ensure that across a week there are opportunities planned within the daily provision for mathematical development and a clear balance between child-initiated and adult-led activities

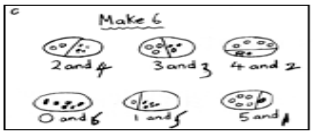
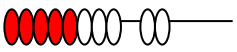
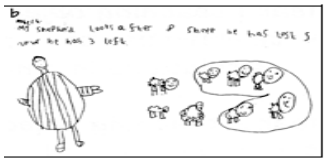
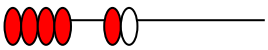
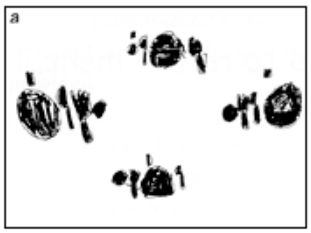
Key Resources for teaching number:

NUMICON as the key visual for number, numbers in the environment

Key resources for counting:

- **Cubes, buttons, threading, cards, magnetic numbers, dominoes, shells, bricks, blocks, fruit, toys, children!**

The table below set out the ***expected*** models and images, and **informal and formal** methods of calculation for teachers to use, model and demonstrate to children at this stage of learning:

Reception			
Addition	**Subtraction**	**Multiplication**	**Division**
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.  **EXPECTATION: all numbers are presented as soon as possible on a line to show the relationship between them** Bead strings or bead bars can be used to illustrate addition  8+2=10 They use numberlines and practical resources to support calculation and teachers *demonstrate* the use of the numberline.  	Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.  Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.  **6-2=4** They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line. **Key point:** there are at least 5 contextual interpretations of subtraction that need to be taught: **1.Partitioning and take away** **2.Comparison** **3.The complement of a set** **4. Reduction, counting back** **5.The inverse of addition** *(See Derek Haylock, Understanding mathematics for Young Children').* **AVOID OVER-EMPHASIS ON 'TAKE-AWAY'**	Children will experience **equal groups** of objects. They will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups. e.g. laying the table for the 3 bears and goldilocks  Begin to introduce children to the visual images of arrays – using real-life examples (brick work, paving slabs, windows in a building, anything with a repeating pattern in rows and columns!)	Children will understand **equal groups** and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.  Count in 2's to find out how many socks are on the washing line: 

Key Stage 1 Progression of calculation

Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

Key Language:
less, more, group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table

Addition and subtraction:
Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations.

A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 – 3 and 15 – 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.
In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.

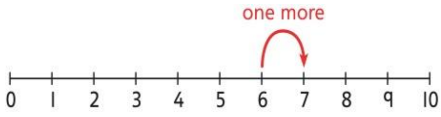
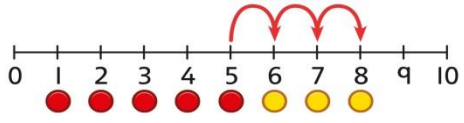
Multiplication and division:
Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division.
They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation.

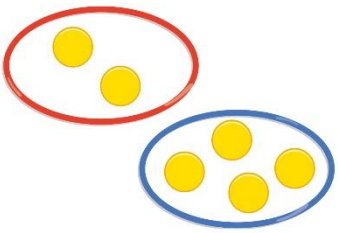
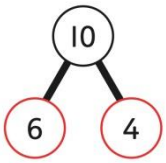
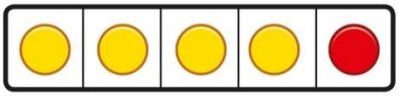
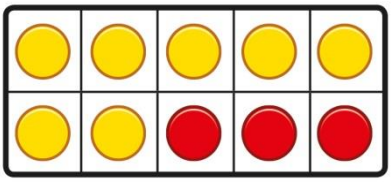
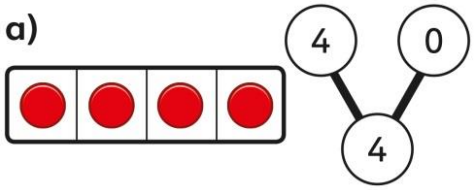
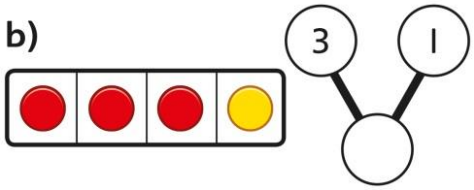
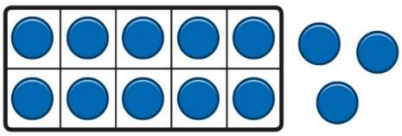
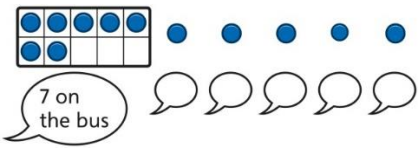
In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations.
Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.

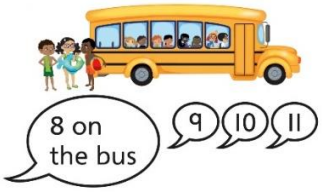
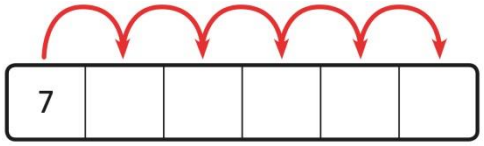
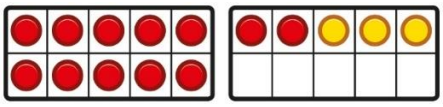
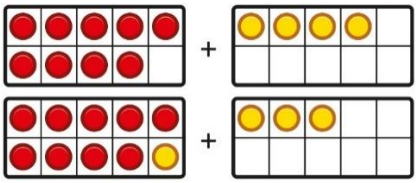
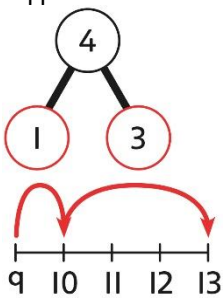
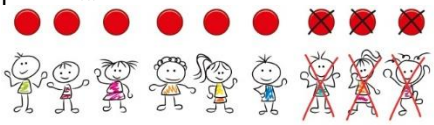
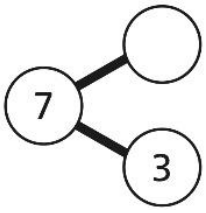
Fractions:
In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole.
In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.

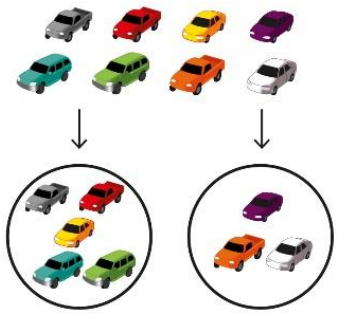
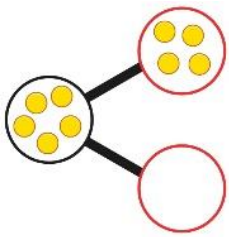
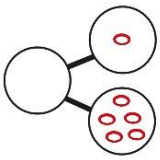
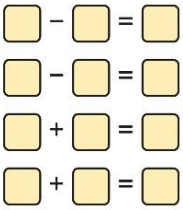
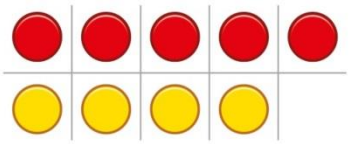
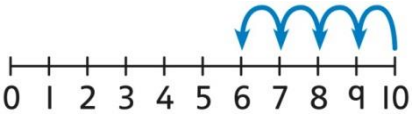
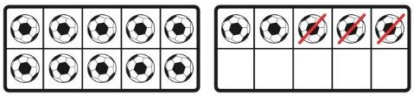
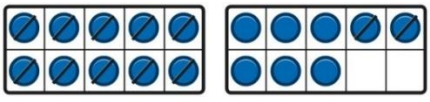
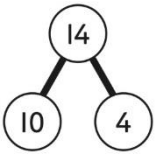
The tables below set out the ***expected*** models and images, and **informal and formal** methods of calculation for teachers to use, model and demonstrate to children at each stage of learning. They focus on a concret, pictorial and abstract teaching approach to embed learning:

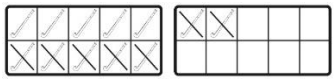
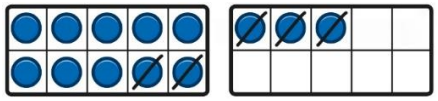
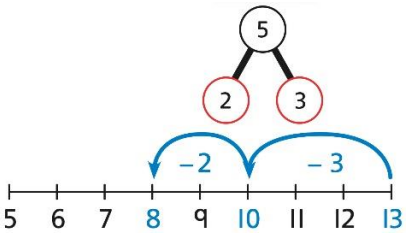
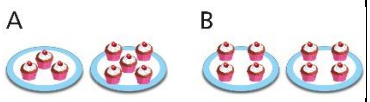
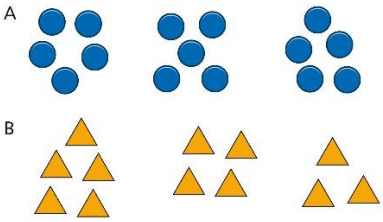
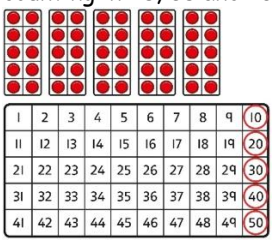
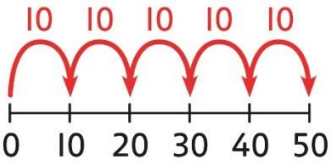
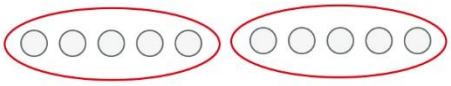
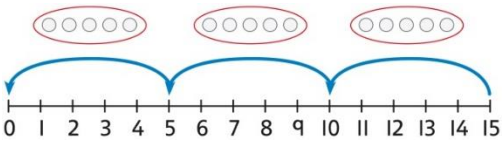
Year 1

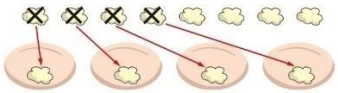
	Concrete	Pictorial	Abstract
Year 1 Addition	**Counting and adding more** Children add one more person or object to a group to find one more.	**Counting and adding more** Children add one more cube or counter to a group to represent one more.  *One more than 4 is 5.*	**Counting and adding more** Use a number line to understand how to link counting on with finding one more.  *One more than 6 is 7.* *7 is one more than 6.* Learn to link counting on with adding more than one.  *5 + 3 = 8*
	Understanding part-part-whole relationship Sort people and objects into parts and understand the relationship with the whole.	**Understanding part-part-whole relationship** Children draw to represent the parts and understand the relationship with the whole.	**Understanding part-part-whole relationship** Use a part-whole model to represent the numbers.

	 *The parts are 2 and 4. The whole is 6.*	 *The parts are 1 and 5. The whole is 6.*	 $\boxed{6} + \boxed{4} = \boxed{10}$ *6 + 4 = 10*
	Knowing and finding number bonds within 10 Break apart a group and put back together to find and form number bonds.  *3 + 4 = 7*  *6 = 2 + 4*	**Knowing and finding number bonds within 10** Use five and ten frames to represent key number bonds.  *5 = 4 + 1*  *10 = 7 + 3*	**Knowing and finding number bonds within 10** Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.   *4 + 0 = 4* *3 + 1 = 4*
	Understanding teen numbers as a complete 10 and some more Complete a group of 10 objects and count more.  *13 is 10 and 3 more.*	**Understanding teen numbers as a complete 10 and some more** Use a ten frame to support understanding of a complete 10 for teen numbers.  *13 is 10 and 3 more.*	**Understanding teen numbers as a complete 10 and some more.** *1 ten and 3 ones equal 13.* *10 + 3 = 13*
	Adding by counting on Children use knowledge of counting to 20 to find a total by counting on using people or objects.	**Adding by counting on** Children use counters to support and represent their counting on strategy.  	**Adding by counting on** Children use number lines or number tracks to support their counting on strategy.

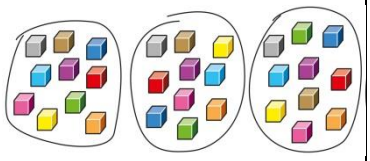
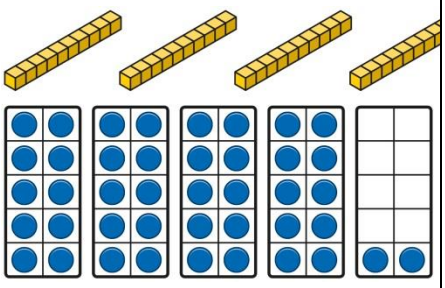
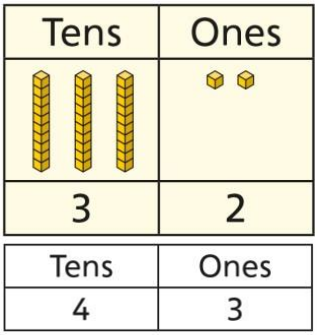
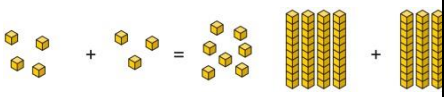
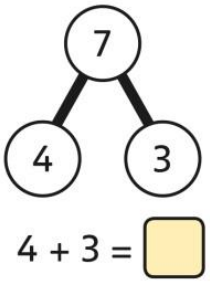
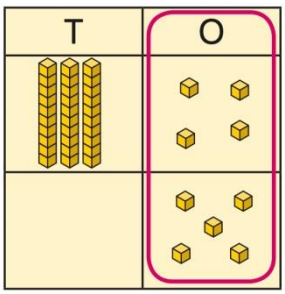
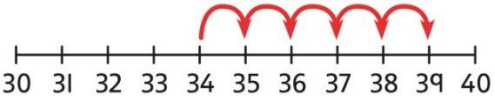
			 $7 + 5 = \square$
	Adding the 1s Children use bead strings to recognise how to add the 1s to find the total efficiently.  *2 + 3 = 5* *12 + 3 = 15*	**Adding the 1s** Children represent calculations using ten frames to add a teen and 1s.  *2 + 3 = 5* *12 + 3 = 15*	**Adding the 1s** Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. *3 + 5 = 8* *So, 13 + 5 = 18*
	Bridging the 10 using number bonds Children use a bead string to complete a 10 and understand how this relates to the addition.  *7 add 3 makes 10.* *So, 7 add 5 is 10 and 2 more.*	**Bridging the 10 using number bonds** Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.  .	**Bridging the 10 using number bonds** Use a part-whole model and a number line to support the calculation.  *9 + 4 = 13*
Year 1 Subtraction	**Counting back and taking away** Children arrange objects and remove to find how many are left.  *1 less than 6 is 5.* *6 subtract 1 is 5.*	**Counting back and taking away** Children draw and cross out or use counters to represent objects from a problem.  $9 - \square = \square$ There are $\square$ children left.	**Counting back and taking away** Children count back to take away and use a number line or number track to support the method.  *9 – 3 = 6*
	Finding a missing part, given a whole and a part Children separate a whole into parts and understand how one part can be found by subtraction.	**Finding a missing part, given a whole and a part** Children represent a whole and a part and understand how to find the missing part by subtraction.	**Finding a missing part, given a whole and a part** Children use a part-whole model to support the subtraction to find a missing part.  *7 – 3 = ?*

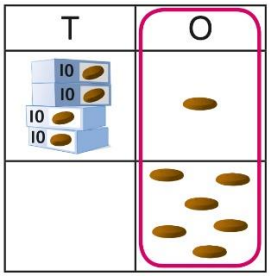
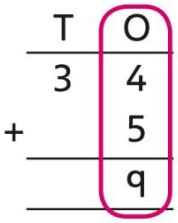
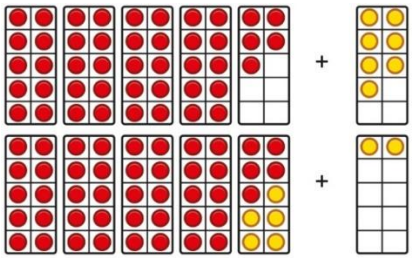
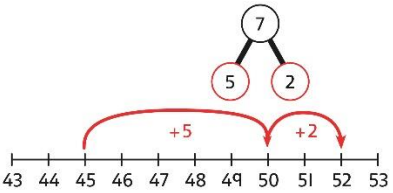
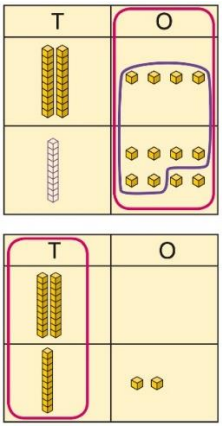
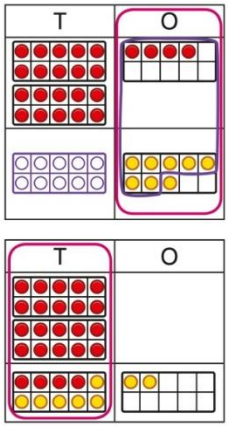
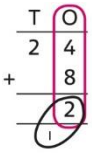
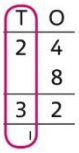
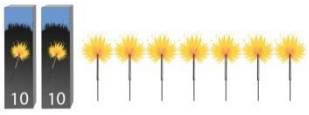
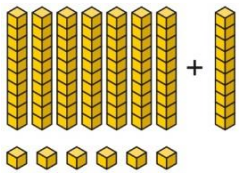
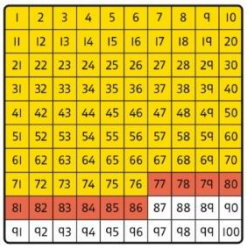
	 *8 – 5 = ?*	 5 – 4 =	Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model.  
	Finding the difference Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than 8.* *The difference between 8 and 6 is 2.*	**Finding the difference** Represent objects using sketches or counters to support finding the difference.  *5 – 4 = 1* *The difference between 5 and 4 is 1.*	**Finding the difference** Children understand 'find the difference' as subtraction.  *10 – 4 = 6* *The difference between 10 and 6 is 4.*
	Subtraction within 20 Understand when and how to subtract 1s efficiently. Use a bead string to subtract 1s efficiently.  *5 – 3 = 2* *15 – 3 = 12*	**Subtraction within 20** Understand when and how to subtract 1s efficiently.  *5 – 3 = 2* *15 – 3 = 12*	**Subtraction within 20** Understand how to use knowledge of bonds within 10 to subtract efficiently. *5 – 3 = 2* *15 – 3 = 12*
	Subtracting 10s and 1s For example: 18 – 12 Subtract 12 by first subtracting the 10, then the remaining 2.  *First subtract the 10, then take away 2.*	**Subtracting 10s and 1s** For example: 18 – 12 Use ten frames to represent the efficient method of subtracting 12.  *First subtract the 10, then subtract 2.*	**Subtracting 10s and 1s** Use a part-whole model to support the calculation.  *19 – 14* *19 – 10 = 9* *9 – 4 = 5* *So, 19 – 14 = 5*
	Subtraction bridging 10 using number bonds	**Subtraction bridging 10 using number bonds**	**Subtraction bridging 10 using number bonds**

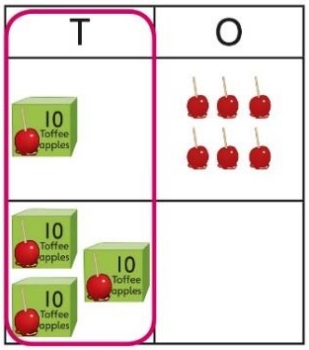
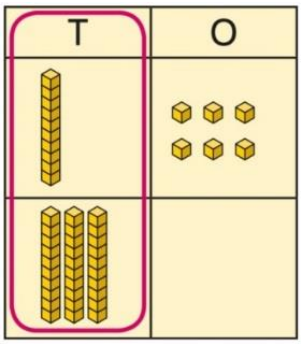
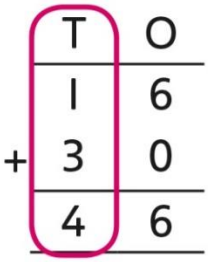
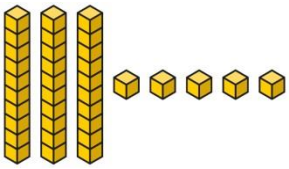
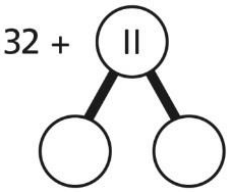
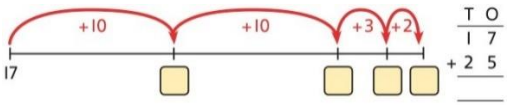
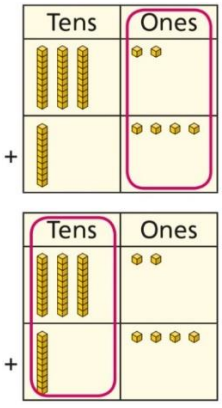
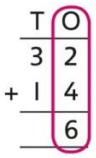
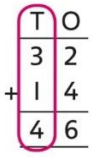
	For example: 12 – 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts.  *7 is 2 and 5, so I take away the 2 and then the 5.*	Represent the use of bonds using ten frames.  *For 13 – 5, I take away 3 to make 10, then take away 2 to make 8.*	Use a number line and a part-whole model to support the method. *13 – 5* 
Year 1 Multiplication	**Recognising and making equal groups** Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal. 	**Recognising and making equal groups** Children draw and represent equal and unequal groups. 	**Describe equal groups using words** *Three equal groups of 4.* *Four equal groups of 3.*
	Finding the total of equal groups by counting in 2s, 5s and 10s  There are 5 pens in each pack … 5…10…15…20…25…30…35…40…	**Finding the total of equal groups by counting in 2s, 5s and 10s** 100 squares and ten frames support counting in 2s, 5s and 10s. 	**Finding the total of equal groups by counting in 2s, 5s and 10s** Use a number line to support repeated addition through counting in 2s, 5s and 10s. 
Year 1 Division	**Grouping** Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups.  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.*	**Grouping** Represent a whole and work out how many equal groups.  *There are 10 in total.* *There are 5 in each group.* *There are 2 groups.*	**Grouping** Children may relate this to counting back in steps of 2, 5 or 10. 
	Sharing Share a set of objects into equal parts and work out how many are in each part.	**Sharing** Sketch or draw to represent sharing into equal parts. This may be related to fractions.	**Sharing** *10 shared into 2 equal groups gives 5 in each group.*

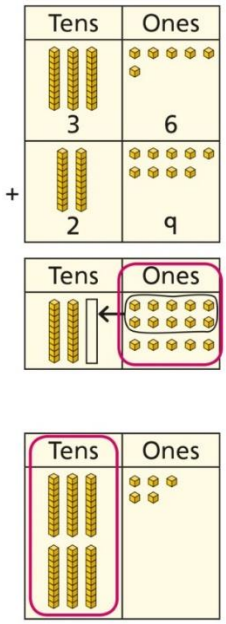
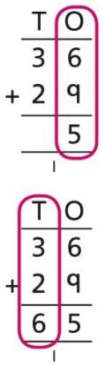
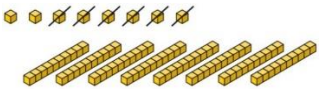
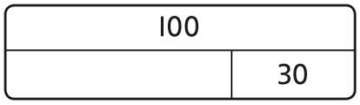
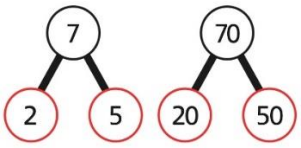
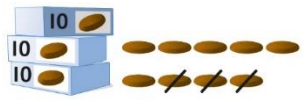
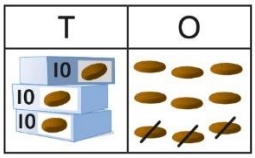
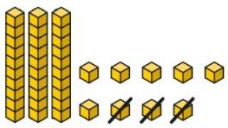
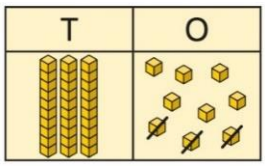
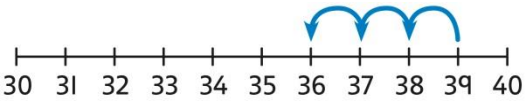
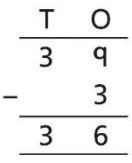
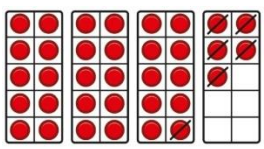
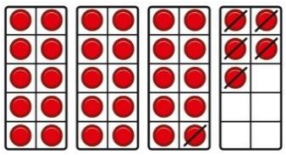
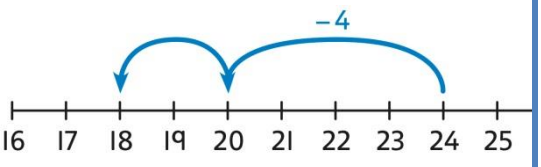
			

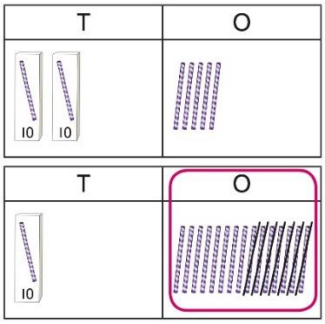
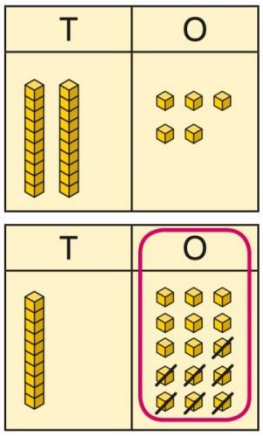
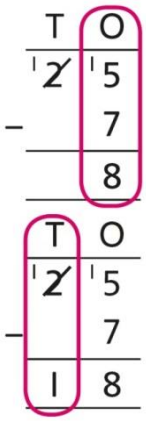
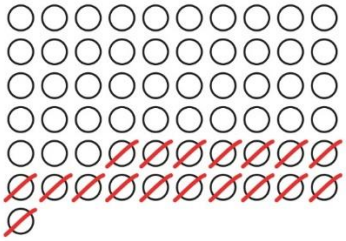
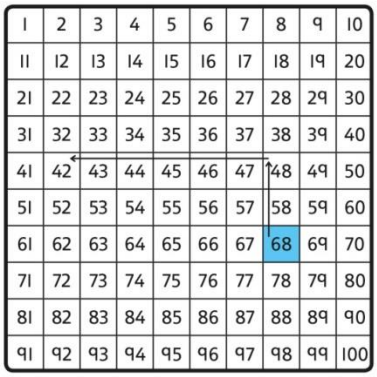
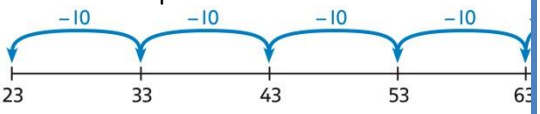
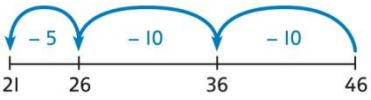
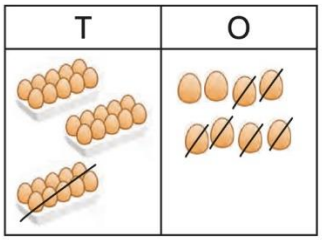
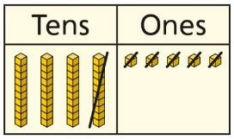
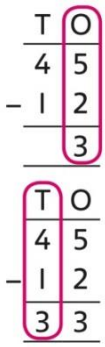
Year 2

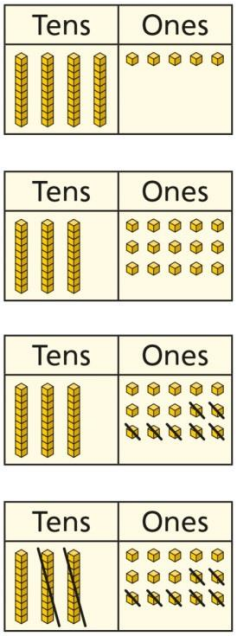
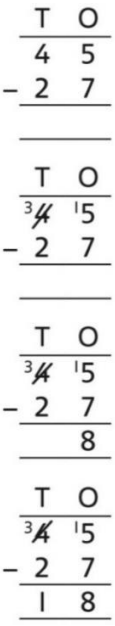
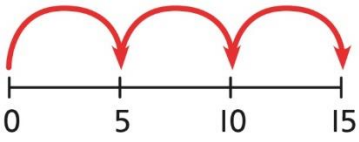
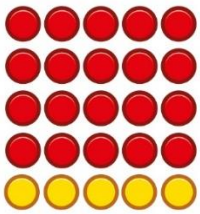
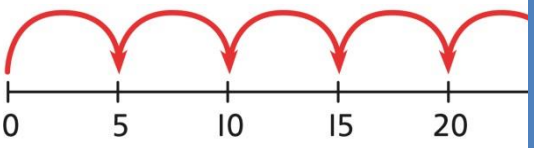
	Concrete	Pictorial	Abstract
Year 2 Addition			
Understanding 10s and 1s	Group objects into 10s and 1s.  Bundle straws to understand unitising of 10s. 	Understand 10s and 1s equipment, and link with visual representations on ten frames. 	Represent numbers on a place value grid, using equipment or numerals.  Tens | Ones 4 | 3
Adding 10s	Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10s.  *4 + 3 = 7* *4 tens + 3 tens = 7 tens* *40 + 30 = 70*
Adding a 1-digit number to a 2-digit number not bridging a 10	Add the 1s to find the total. Use known bonds within 10.  *41 is 4 tens and 1 one.* *41 add 6 ones is 4 tens and 7 ones.* This can also be done in a place value grid.	Add the 1s.  *34 is 3 tens and 4 ones.* *4 ones and 5 ones are 9 ones.* *The total is 3 tens and 9 ones.*  	Add the 1s. Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.  This can be represented horizontally or vertically. *34 + 5 = 39* *or*

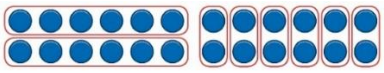
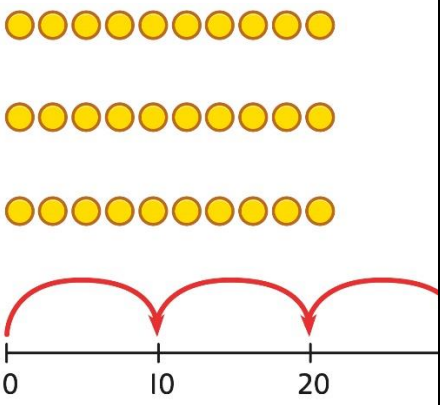
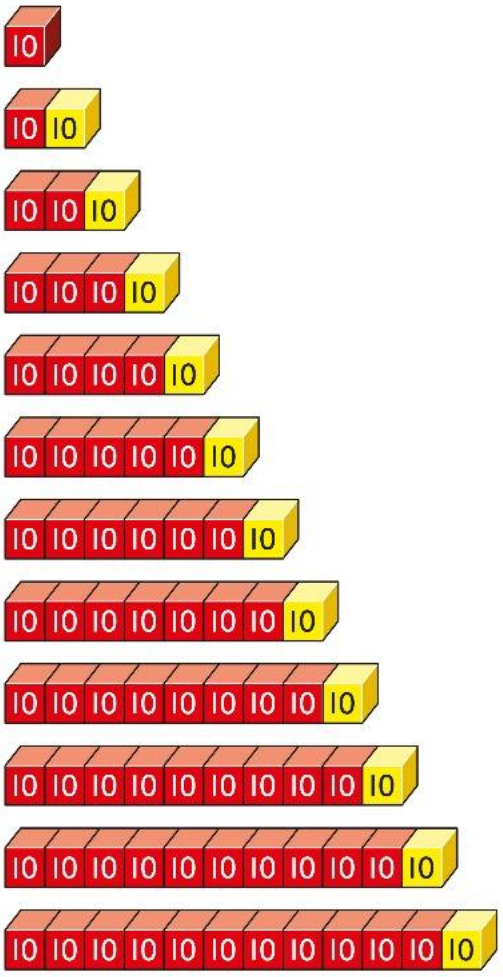
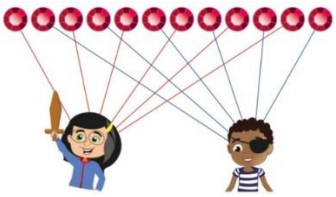
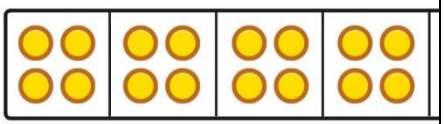
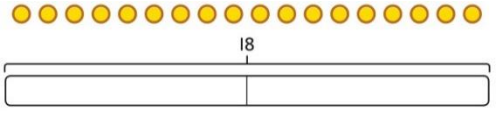
			
Adding a 1-digit number to a 2-digit number bridging 10	Complete a 10 using number bonds.  *There are 4 tens and 5 ones.* *I need to add 7. I will use 5 to complete a 10, then add 2 more.*	Complete a 10 using number bonds. 	Complete a 10 using number bonds.  *7 = 5 + 2* *45 + 5 + 2 = 52*
Adding a 1-digit number to a 2-digit number using exchange	Exchange 10 ones for 1 ten. 	Exchange 10 ones for 1 ten. 	Exchange 10 ones for 1 ten.  
Adding a multiple of 10 to a 2-digit number	Add the 10s and then recombine.  *27 is 2 tens and 7 ones.* *50 is 5 tens.* *There are 7 tens in total and 7 ones.* *So, 27 + 50 is 7 tens and 7 ones.*	Add the 10s and then recombine.  *66 is 6 tens and 6 ones.* *66 + 10 = 76* A 100 square can support this understanding. 	Add the 10s and then recombine. *37 + 20 = ?* *30 + 20 = 50* *50 + 7 = 57* *37 + 20 = 57*

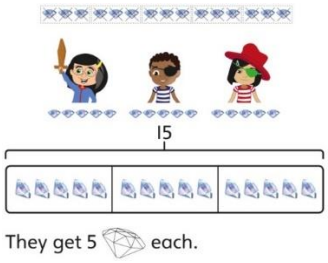
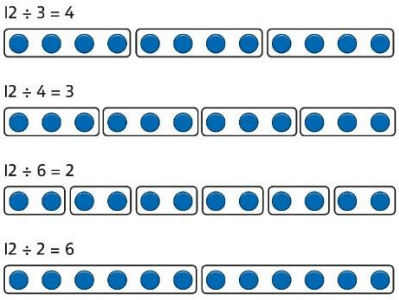
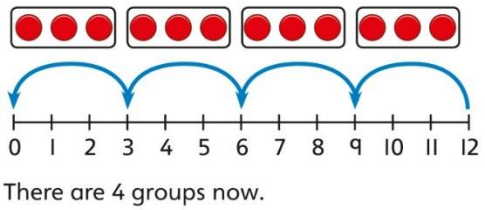
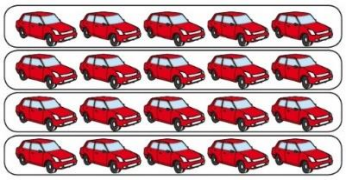
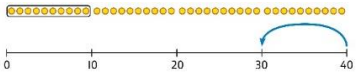
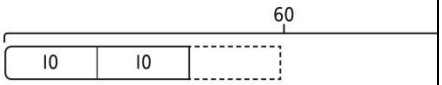
Adding a multiple of 10 to a 2-digit number using columns	Add the 10s using a place value grid to support.  *16 is 1 ten and 6 ones.* *30 is 3 tens.* *There are 4 tens and 6 ones in total.*	Add the 10s using a place value grid to support.  *16 is 1 ten and 6 ones.* *30 is 3 tens.* *There are 4 tens and 6 ones in total.*	Add the 10s represented vertically. Children must understand how the method relates to unitising of 10s and place value.  *1 + 3 = 4* *1 ten + 3 tens = 4 tens* *16 + 30 = 46*
Adding two 2-digit numbers	Add the 10s and 1s separately.  *5 + 3 = 8* *There are 8 ones in total.* *3 + 2 = 5* *There are 5 tens in total.* *35 + 23 = 58*	Add the 10s and 1s separately. Use a part-whole model to support.  *11 = 10 + 1* *32 + 10 = 42* *42 + 1 = 43* *32 + 11 = 43*	Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations.  *17 + 25*
Adding two 2-digit numbers using a place value grid	Add the 1s. Then add the 10s.  		Add the 1s. Then add the 10s.  
Adding two 2-digit numbers with exchange	Add the 1s. Exchange 10 ones for a ten. Then add the 10s.		Add the 1s. Exchange 10 ones for a ten. Then add the 10s.

			
Year 2 Subtraction			
Subtracting multiples of 10	Use known number bonds and unitising to subtract multiples of 10.  *8 subtract 6 is 2.* *So, 8 tens subtract 6 tens is 2 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *10 – 3 = 7* *So, 10 tens subtract 3 tens is 7 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *7 tens subtract 5 tens is 2 tens.* *70 – 50 = 20*
Subtracting a single-digit number	Subtract the 1s. This may be done in or out of a place value grid.  	Subtract the 1s. This may be done in or out of a place value grid.  	Subtract the 1s. Understand the link between counting back and subtracting the 1s using known bonds.   *9 – 3 = 6* *39 – 3 = 36*
Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds.  *35 – 6*	Bridge 10 by using known bonds.  *35 – 6* *First, I will subtract 5, then 1.*	Bridge 10 by using known bonds.  *24 – 6 = ?* *24 – 4 – 2 = ?*

	I took away 5 counters, then 1 more.		
Subtracting a single-digit number using exchange	Exchange 1 ten for 10 ones. This may be done in or out of a place value grid.  	Exchange 1 ten for 10 ones.  	Exchange 1 ten for 10 ones.  *25 – 7 = 18*
Subtracting a 2-digit number	Subtract by taking away.  *61 – 18* *I took away 1 ten and 8 ones.*	Subtract the 10s and the 1s. This can be represented on a 100 square.  	Subtract the 10s and the 1s. This can be represented on a number line.  *64 – 41 = ?* *64 – 1 = 63* *63 – 40 = 23* *64 – 41 = 23*  *46 – 20 = 26* *26 – 5 = 21* *46 – 25 = 21*
Subtracting a 2-digit number using place value and columns	Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid.  *38 – 16 = 22*	Subtract the 1s. Then subtract the 10s.  	Using column subtraction, subtract the 1s. Then subtract the 10s. 

Subtracting a 2-digit number with exchange		Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. 	Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. 
Year 2 Multiplication			
Equal groups and repeated addition	Recognise equal groups and write as repeated addition and as multiplication.  *3 groups of 5 chairs* *15 chairs altogether*	Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.  *3 groups of 5* *15 in total*	Use a number line and write as repeated addition and as multiplication.  *5 + 5 + 5 = 15* *3 × 5 = 15*
Using arrays to represent multiplication and support understanding	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5*	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5 … 5 groups of 5*	Understand the relationship between arrays, multiplication and repeated addition.  *5 × 5 = 25*
Understanding commutativity	Use arrays to visualise commutativity.	Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.	Use arrays to visualise commutativity.

	 *I can see 6 groups of 3.* *I can see 3 groups of 6.*	 *This is 2 groups of 6 and also 6 groups of 2.*	 *4 + 4 + 4 + 4 + 4 = 20* *5 + 5 + 5 + 5 = 20* *4 × 5 = 20 and 5 × 4 = 20*
Learning ×2, ×5 and ×10 table facts	Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.  *3 groups of 10 … 10, 20, 30* *3 × 10 = 30*	Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.  *10 + 10 + 10 = 30* *3 × 10 = 30*	Understand how the times-tables increase and contain patterns.  *5 × 10 = 50* *6 × 10 = 60*
Year 2 Division			
Sharing equally	Start with a whole and share into equal parts, one at a time. 	Represent the objects shared into equal parts using a bar model. 	Use a bar model to support understanding of the division. 

	12 shared equally between 2. They get 6 each. Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared  *15 shared equally between 3. They get 5 each.*	*20 shared into 5 equal parts. There are 4 in each part.*	*18 ÷ 2 = 9*
Grouping equally	Understand how to make equal groups from a whole.  *8 divided into 4 equal groups. There are 2 in each group.*	Understand the relationship between grouping and the division statements.  	Understand how to relate division by grouping to repeated subtraction.  *12 divided into groups of 3.* *12 ÷ 3 = 4* *There are 4 groups.*
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  *4 groups of 5 cars is 20 cars in total.* *20 divided by 4 is 5.*	Link equal grouping with repeated subtraction and known times-table facts to support division.  *40 divided by 4 is 10.* Use a bar model to support understanding of the link between times-table knowledge and division.  	Relate times-table knowledge directly to division. 1 × 10 = 10 2 × 10 = 20 **3 × 10 = 30** 4 × 10 = 40 5 × 10 = 50 6 × 10 = 60 7 × 10 = 70 8 × 10 = 80 I used the 10 times-table to help me. 3 × 10 = 30. *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*

Key stage 2

In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping.

Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply.

In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns.

By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

Multiplication and division: Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.

Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively.

Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and

problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit.

Children develop column methods to support multiplications in these cases.

For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts.

Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

Fractions: Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside.

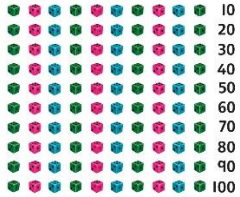
in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1.

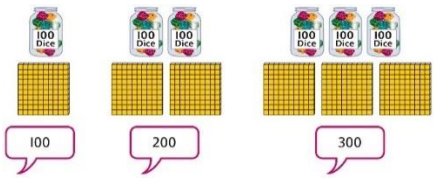
Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

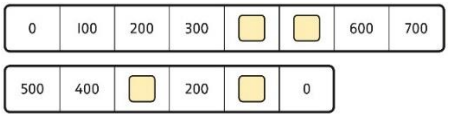
The tables below set out the expected models and images, and informal and formal methods of calculation for teachers to use, model and demonstrate to children at each stage of learning

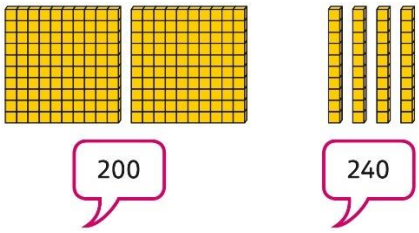
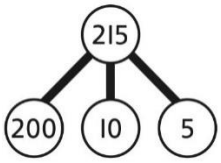
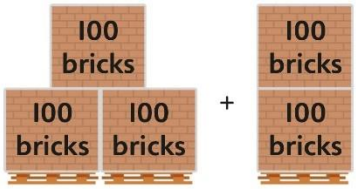
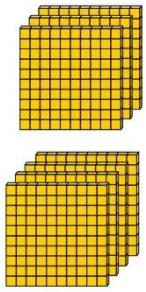
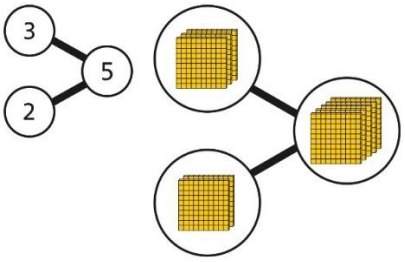
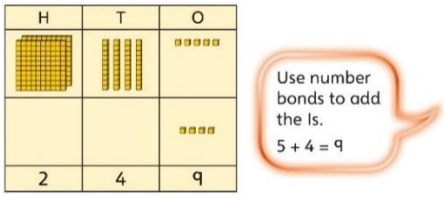
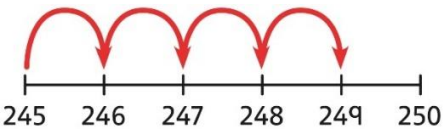
<table>
<tr><th colspan="3">Year 3</th></tr>
<tr><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Understand the cardinality of 100, and the link with 10 tens.

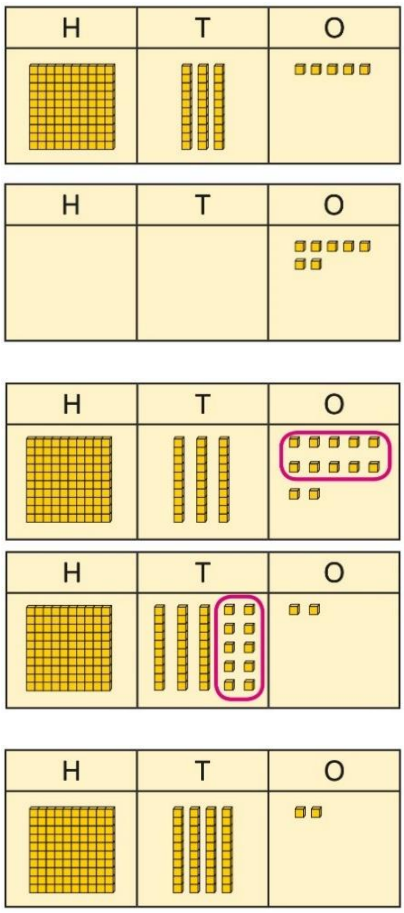
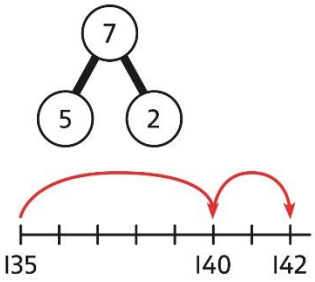
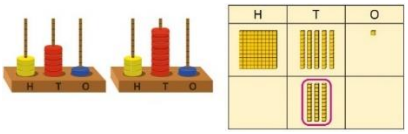
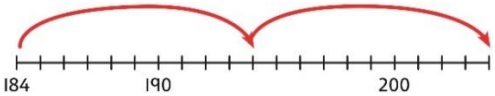
Use cubes to place into groups of 10 tens.

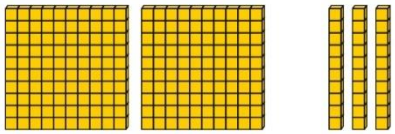
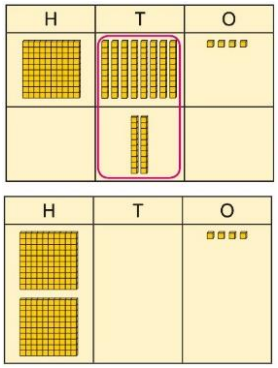
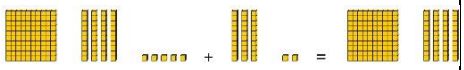
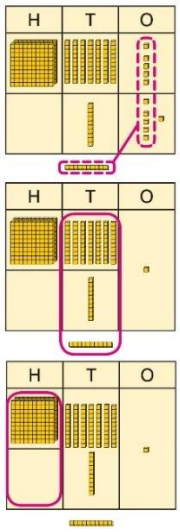
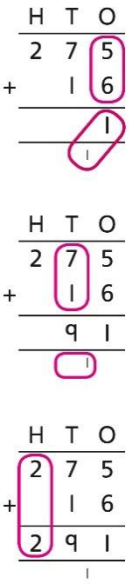
 
</td>
<td>Unitise 100 and count in steps of 100.

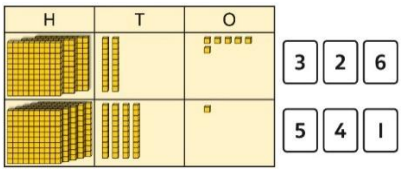
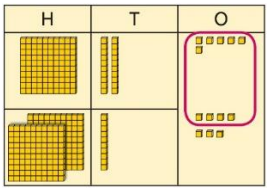
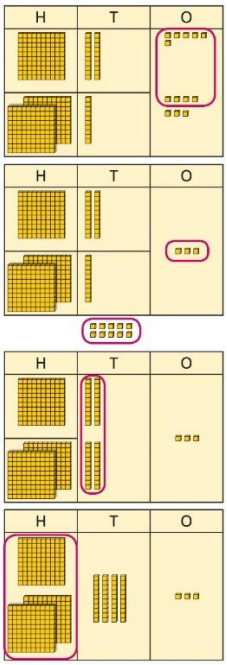
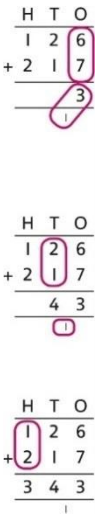
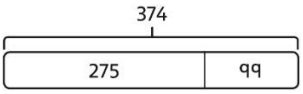
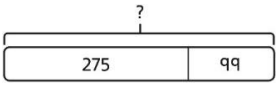
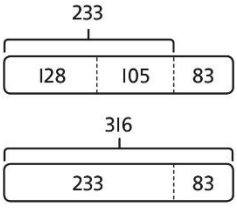
 
</td>
<td>Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0.

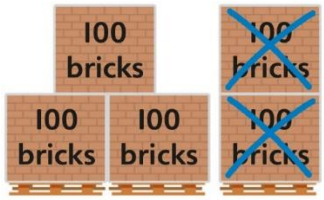
 
</td>
</tr>
<tr>
<td>Unitise 100s, 10s and 1s to build 3-digit numbers.</td>
<td>Use equipment to represent numbers to 1,000.</td>
<td>Represent the parts of numbers to 1,000 using a part-whole model.</td>
</tr>
</table>

	 Use a place value grid to support the structure of numbers to 1,000. Place value counters are used alongside other equipment. Children should understand how each counter represents a different unitised amount.	 *215 = 200 + 10 + 5* Recognise numbers to 1,000 represented on a number line, including those between intervals.
Use known facts and unitising to add multiples of 100.  *3 + 2 = 5* *3 hundreds + 2 hundreds = 5 hundreds* *300 + 200 = 500*	Use known facts and unitising to add multiples of 100.  *3 + 4 = 7* *3 hundreds + 4 hundreds = 7 hundreds* *300 + 400 = 700*	Use known facts and unitising to add multiples of 100. Represent the addition on a number line. Use a part-whole model to support unitising.  *3 + 2 = 5* *300 + 200 = 500*
Use number bonds to add the 1s.  *214 + 4 = ?* *Now there are 4 + 4 ones in total.* *4 + 4 = 8* *214 + 4 = 218*	Use number bonds to add the 1s.  *245 + 4* *5 + 4 = 9* *245 + 4 = 249*	Understand the link with counting on. *245 + 4*  Use number bonds to add the 1s and understand that this is more efficient and less prone to error. *245 + 4 = ?* *I will add the 1s.* *5 + 4 = 9* *So, 245 + 4 = 249*
Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten.	Exchange 10 ones for 1 ten where needed. Use a place value grid to support the understanding.	Understand how to bridge by partitioning to the 1s to make the next 10.

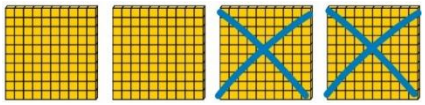
Children should explore this using unitised objects or physical apparatus.	 *135 + 7 = 142*	 *135 + 7 = ?* *135 + 5 + 2 = 142* Ensure that children understand how to add 1s bridging a 100. *198 + 5 = ?* *198 + 2 + 3 = 203*
Calculate mentally by forming the number bond for the 10s.  *234 + 50* *There are 3 tens and 5 tens altogether.* *3 + 5 = 8* *In total there are 8 tens.* *234 + 50 = 284*	Calculate mentally by forming the number bond for the 10s. *351 + 30 = ?*  *5 tens + 3 tens = 8 tens* *351 + 30 = 381*	Calculate mentally by forming the number bond for the 10s. *753 + 40* *I know that 5 + 4 = 9* *So, 50 + 40 = 90* *753 + 40 = 793*
Understand the exchange of 10 tens for 1 hundred.	Add by exchanging 10 tens for 1 hundred. *184 + 20 = ?*	Understand how the addition relates to counting on in 10s across 100. 

	 *184 + 20 = 204*	*184 + 20 = ?* *I can count in 10s … 194 … 204* *184 + 20 = 204* Use number bonds within 20 to support efficient mental calculations. *385 + 50* *There are 8 tens and 5 tens.* *That is 13 tens.* *385 + 50 = 300 + 130 + 5* *385 + 50 = 435*
Use place value equipment to make and combine groups to model addition. 	Use a place value grid to organise thinking and adding of 1s, then 10s.	Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.
Use place value equipment to model addition and understand where exchange is required. *Use place value counters to represent 154 + 72.* *Use this to decide if any exchange is required.* *There are 5 tens and 7 tens. That is 12 tens so I will exchange.*	Represent the required exchange on a place value grid using equipment. *275 + 16 = ?*  *275 + 16 = 291* Note: In this example, a mental method may be more efficient. The numbers for the example calculation have been chosen to allow children to visualise the concept and see how the method relates to place value. Children should be encouraged at every stage to select methods that are accurate and efficient.	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.  *275 + 16 = 291*
Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid. *326 + 541 is represented as:*	Represent the place value grid with equipment to model the stages of column addition.	Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.

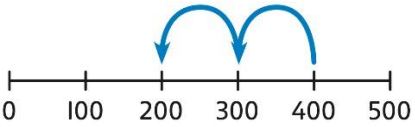
		
Use place value equipment to enact the exchange required.  *There are 13 ones.* *I will exchange 10 ones for 1 ten.*	Model the stages of column addition using place value equipment on a place value grid. 	Use column addition, ensuring understanding of place value at every stage of the calculation.  *126 + 217 = 343* Note: Children should also study examples where exchange is required in more than one column, for example *185 + 318 = ?*
Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 = ?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.* 128 + 105 = 233 
Use known facts and unitising to subtract multiples of 100.	Use known facts and unitising to subtract multiples of 100.	Understand the link with counting back in 100s.



5 – 2 = 3
500 – 200 = 300



4 – 2 = 2
400 – 200 = 200



400 – 200 = 200

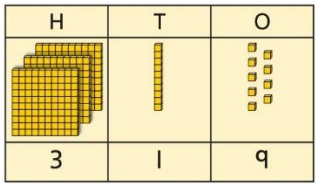
Use known facts and unitising as efficient and accurate methods.
I know that 7 – 4 = 3. Therefore, I know that 700 – 400 = 300.

Use number bonds to subtract the 1s.

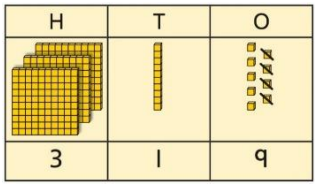
214 – 3 = ?

4 – 3 = 1
214 – 3 = 211

Use number bonds to subtract the 1s.



319 – 4 = ?

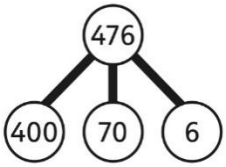


9 – 4 = 5
319 – 4 = 315

Understand the link with counting back using a number line.

Use known number bonds to calculate mentally.

476 – 4 = ?



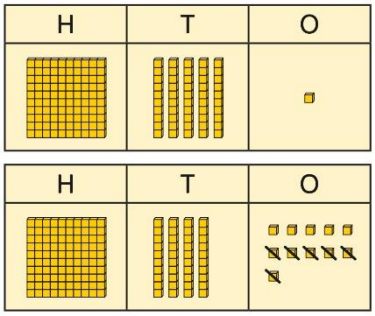
6 – 4 = 2
476 – 4 = 472

Understand why an exchange is necessary by exploring why 1 ten must be exchanged.

Use place value equipment.

Represent the required exchange on a place value grid.

151 – 6 = ?

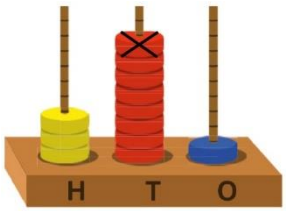


Calculate mentally by using known bonds.

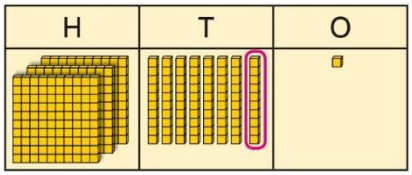
151 – 6 = ?

151 – 1 – 5 = 145

Subtract the 10s using known bonds.



Subtract the 10s using known bonds.



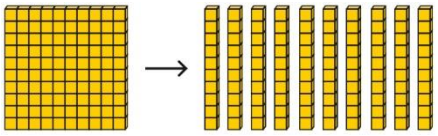
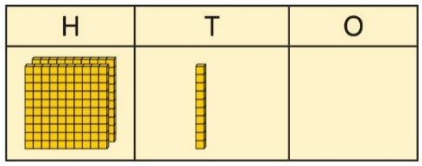
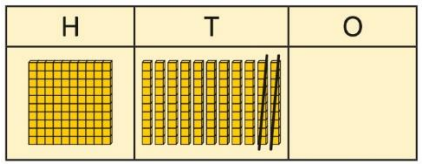
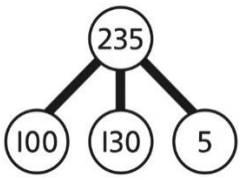
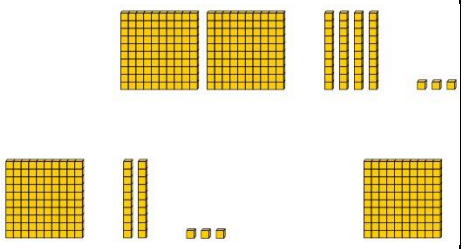
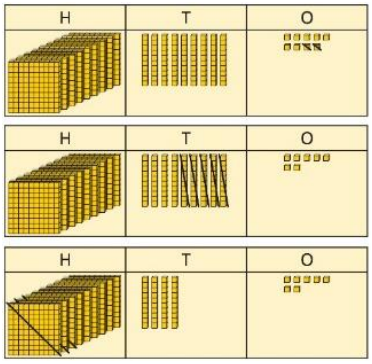
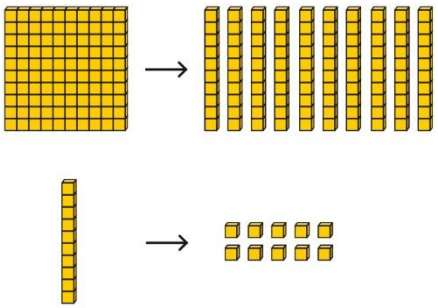
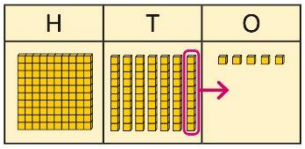
8 tens – 1 ten = 7 tens
381 – 10 = 371

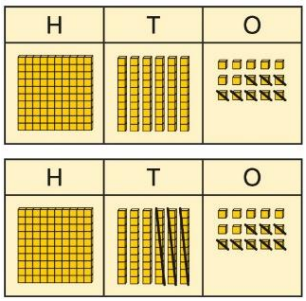
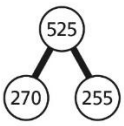
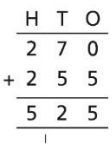
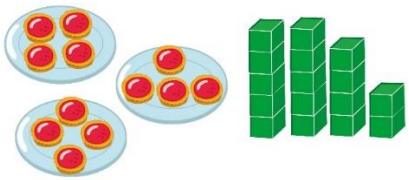
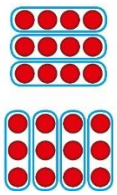
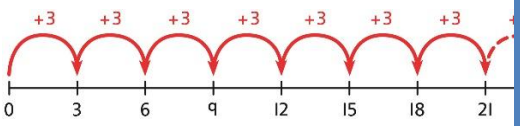
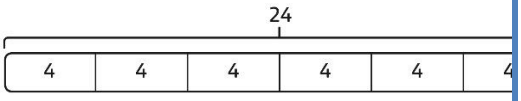
Use known bonds to subtract the 10s mentally.

372 – 50 = ?

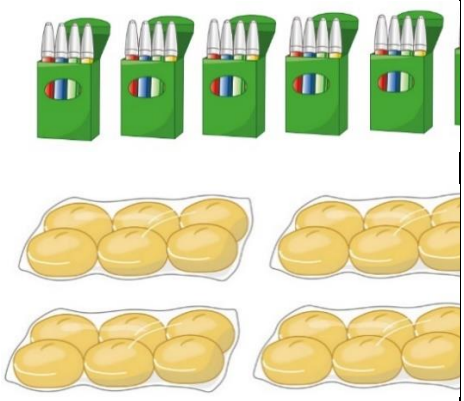
70 – 50 = 20

So, 372 – 50 = 322

381 – 10 = ? *8 tens with 1 removed is 7 tens.* *381 – 10 = 371*		
Use equipment to understand the exchange of 1 hundred for 10 tens. 	Represent the exchange on a place value grid using equipment. *210 – 20 = ?*  *I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.*  *210 – 20 = 190*	Understand the link with counting back on a number line. Use flexible partitioning to support the calculation. *235 – 60 = ?*  *235 = 100 + 130 + 5* *235 – 60 = 100 + 70 + 5* *= 175*
Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away. 	Represent the calculation on a place value grid.  	Use column subtraction to calculate accurately and efficiently. H T O 9 9 9 – 3 5 2 7 H T O 9 9 9 – 3 5 2 4 7 H T O 9 9 9 – 3 5 2 6 4 7
Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones. 	*Model the required exchange on a place value grid.* *175 – 38 = ?* *I need to subtract 8 ones, so I will exchange a ten for 10 ones.*  	Use column subtraction to work accurately and efficiently. H T O 1 6 15 – 3 8 1 3 7 175 – 38 = 137 If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly.

		Children should also understand how to exchange in calculations where there is a zero in the 10s column. 
	Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.  Bar models can also be used to show that a part must be taken away from the whole.	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding. *I have completed this subtraction.* *525 – 270 = 255* *I will check using addition.*  
Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and non-examples using objects.  Children recognise that arrays can be used to model commutative multiplications.  *I can see 3 groups of 8.* *I can see 8 groups of 3.*	Children recognise that arrays demonstrate commutativity.  *This is 3 groups of 4.* *This is 4 groups of 3.*	Children understand the link between repeated addition and multiplication.  *8 groups of 3 is 24.* *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24* A bar model may represent multiplications as equal groups.  *6 × 4 = 24*

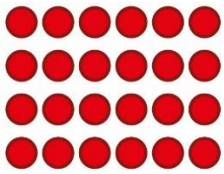
Understand how to use times-tables facts flexibly.



There are 6 groups of 4 pens.
There are 4 groups of 6 bread rolls.

I can use 6 × 4 = 24 to work out both totals.

Understand how times-table facts relate to commutativity.



6 × 4 = 24
4 × 6 = 24

Understand how times-table facts relate to commutativity.

I need to work out 4 groups of 7.

I know that 7 × 4 = 28

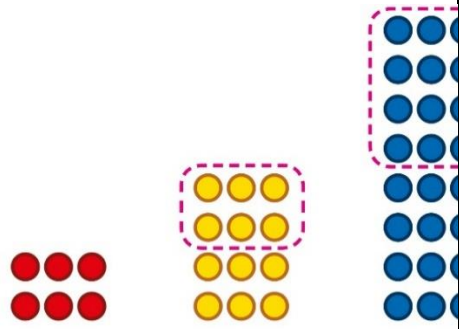
so, I know that

4 groups of 7 = 28
and
7 groups of 4 = 28.

Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.

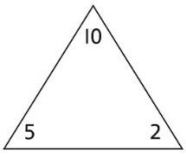
I can use the ×3 table to work out how many keys.
I can also use the ×3 table to work out how many batteries.

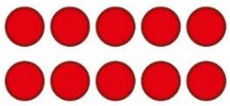
Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.



3 × 2 = 6 3 × 4 = 12 3 × 8 =

Children understand the relationship between related multiplication and division facts in known times-tables.



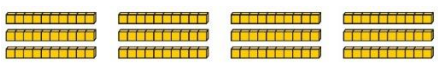


2 × 5 = 10
5 × 2 = 10
10 ÷ 5 = 2
10 ÷ 2 = 5

Explore the relationship between known times-tables and multiples of 10 using place value equipment.

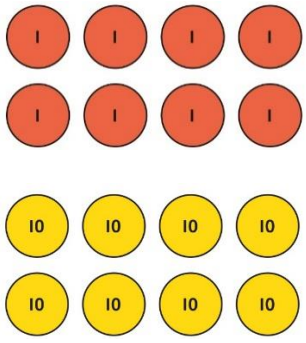
Make 4 groups of 3 ones.

Make 4 groups of 3 tens.



What is the same?
What is different?

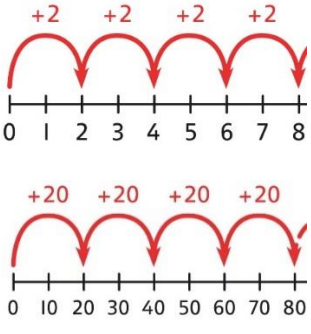
Understand how unitising 10s supports multiplying by multiples of 10.



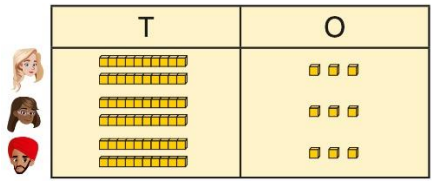
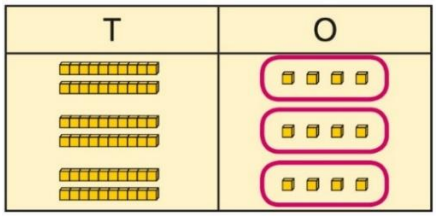
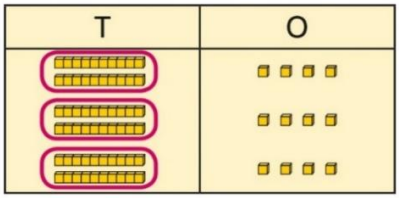
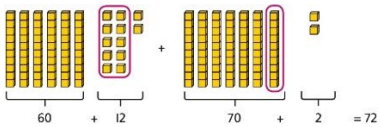
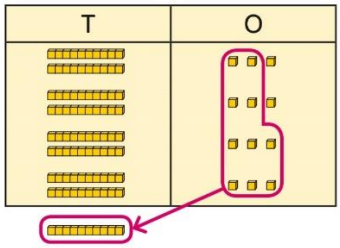
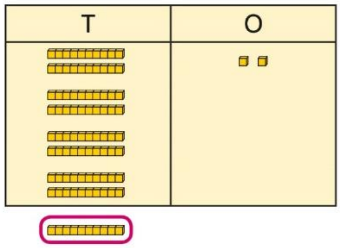
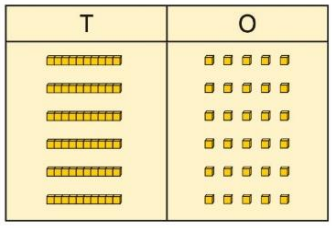
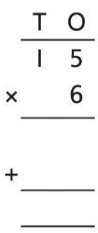
4 groups of 2 ones is 8 ones.
4 groups of 2 tens is 8 tens.

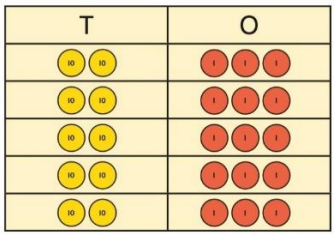
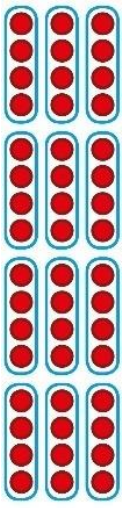
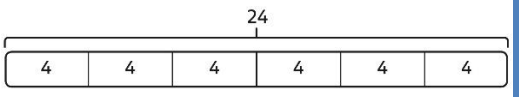
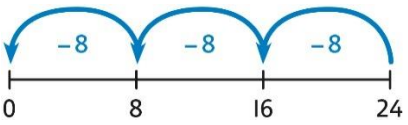
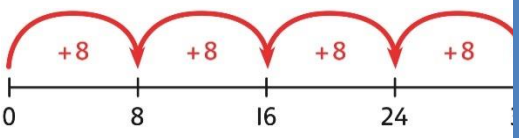
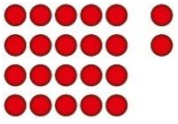
4 × 2 = 8

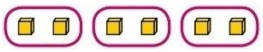
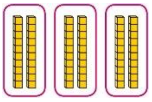
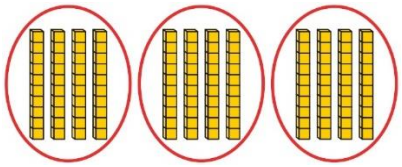
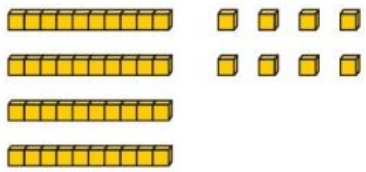
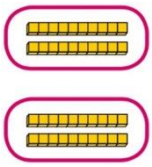
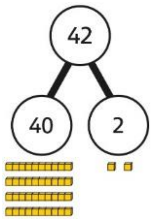
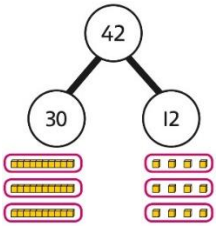
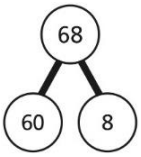
Understand how to use known times-tables to multiply multiples of 10.



4 × 2 = 8
4 × 20 = 80

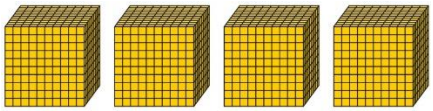
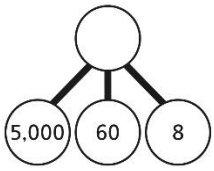
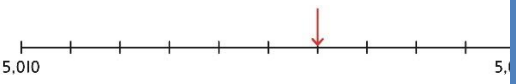
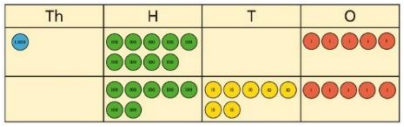
	4 × 20 = 80	
Understand how to link partitioning a 2-digit number with multiplying. *Each person has 23 flowers.* *Each person has 2 tens and 3 ones.*  *There are 3 groups of 2 tens.* *There are 3 groups of 3 ones.* Use place value equipment to model the multiplication context.  *There are 3 groups of 3 ones.* *There are 3 groups of 2 tens.*	Use place value to support how partitioning is linked with multiplying by a 2-digit number. *3 × 24 = ?*  *3 × 4 = 12*  *3 × 20 = 60* *60 + 12 = 72* *3 × 24 = 72*	Use addition to complete multiplications of 2-digit numbers by a 1-digit number. *4 × 13 = ?* *4 × 3 = 12* *4 × 10 = 40* *12 + 40 = 52* *4 × 13 = 52*
Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications. *3 × 24 = ?* *3 × 20 = 60* *3 × 4 = 12*  *3 × 24 = 60 + 12* *3 × 24 = 70 + 2* *3 × 24 = 72*	Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s. *4 × 23 = ?*   	Children may write calculations in expanded column form, but must understand the link with place value and exchange. Children are encouraged to write the expanded parts of the calculation separately.   *5 × 28 = ?*

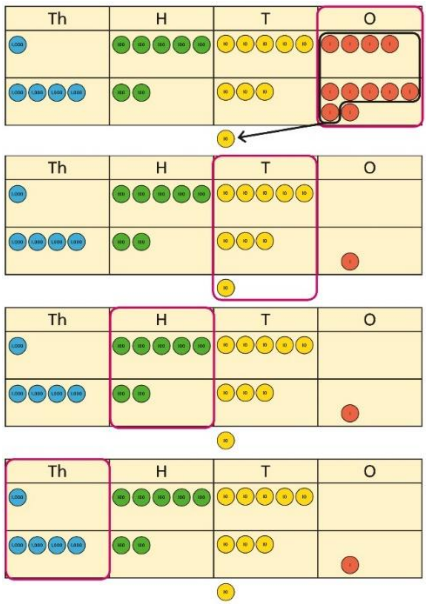
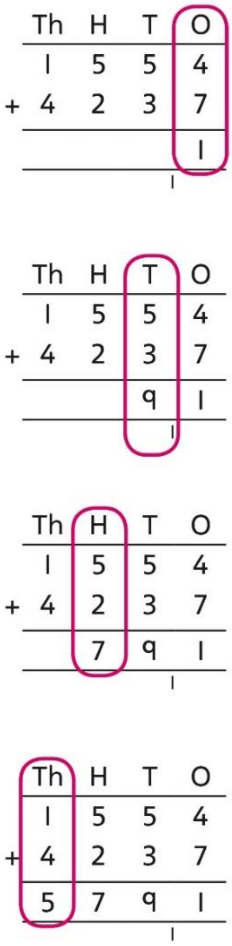
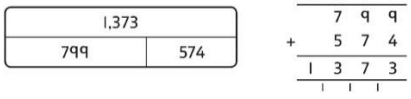
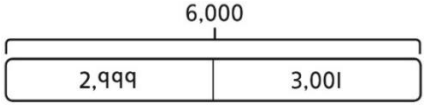
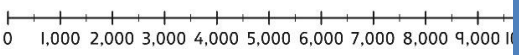
	4 × 23 = 92  *5 × 23 = ?* *5 × 3 = 15* *5 × 20 = 100* *5 × 23 = 115*	T O 2 8 × 5 4 0 5 × 8 1 0 0 5 × 20 1 4 0
Use knowledge of known times-tables to calculate divisions.  *24 divided into groups of 8.* *There are 3 groups of 8.*	Use knowledge of known times-tables to calculate divisions.  48 ÷ 4 = 12 *48 divided into groups of 4.* *There are 12 groups.* *4 × 12 = 48* *48 ÷ 4 = 12*	Use knowledge of known times-tables to calculate divisions. *I need to work out 30 shared between 5.* *I know that 6 × 5 = 30* *so I know that 30 ÷ 5 = 6.* A bar model may represent the relationship between sharing and grouping.  *24 ÷ 4 = 6* *24 ÷ 6 = 4* Children understand how division is related to both repeated subtraction and repeated addition.  *24 ÷ 8 = 3*  *32 ÷ 8 = 4*
Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further. 	Use images to explain remainders.  *22 ÷ 5 = 4 remainder 2*	Understand that the remainder is what cannot be shared equally from a set. *22 ÷ 5 = ?* *3 × 5 = 15* *4 × 5 = 20* *5 × 5 = 25 … this is larger than 22*

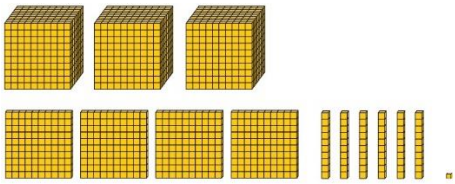
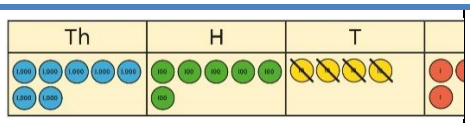
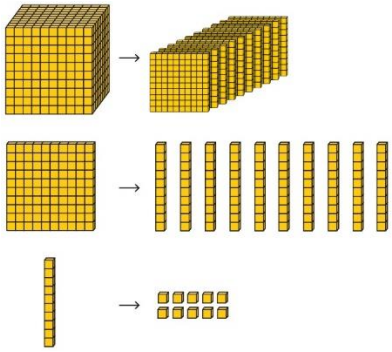
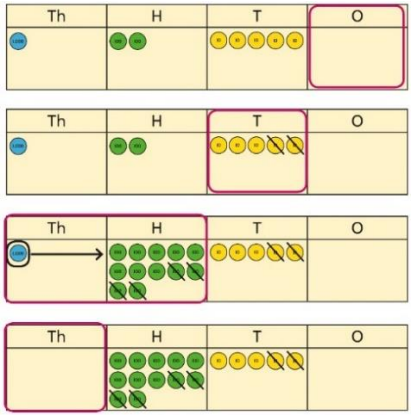
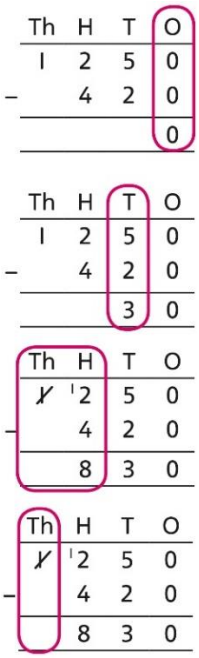
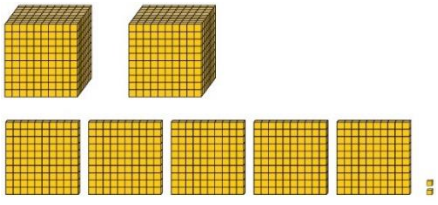
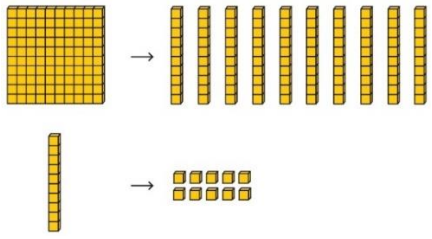
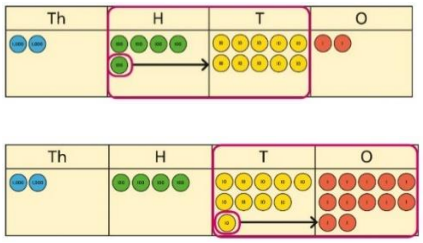
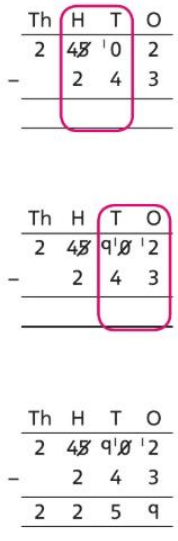
There are 13 sticks in total. *There are 3 groups of 4, with 1 remainder.*		*So, 22 ÷ 5 = 4 remainder 2*
Use place value equipment to understand how to divide by unitising. *Make 6 ones divided by 3.*  *Now make 6 tens divided by 3.*  *What is the same? What is different?*	Divide multiples of 10 by unitising.  *12 tens shared into 3 equal groups.* *4 tens in each group.*	Divide multiples of 10 by a single digit using known times-tables. *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*
Children explore dividing 2-digit numbers by using place value equipment.  *48 ÷ 2 = ?* *First divide the 10s.*  *Then divide the 1s.* 	Children explore which partitions support particular divisions.  *I need to partition 42 differently to divide by 3.*  *42 = 30 + 12* *42 ÷ 3 = 14*	Children partition a number into 10s and 1s to divide where appropriate.  *60 ÷ 2 = 30* *8 ÷ 2 = 4* *30 + 4 = 34* *68 ÷ 2 = 34* Children partition flexibly to divide where appropriate. *42 ÷ 3 = ?* *42 = 40 + 2* *I need to partition 42 differently to divide by 3.* *42 = 30 + 12* *30 ÷ 3 = 10* *12 ÷ 3 = 4* *10 + 4 = 14* *42 ÷ 3 = 14*
Use place value equipment to understand the concept of remainder. *Make 29 from place value equipment.* *Share it into 2 equal groups.* 	Use place value equipment to understand the concept of remainder in division. *29 ÷ 2 = ?*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2*

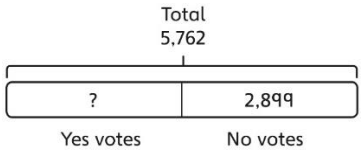
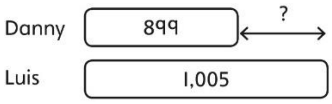
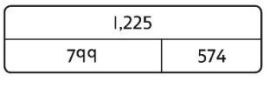
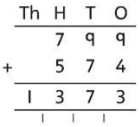
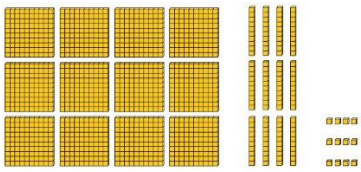
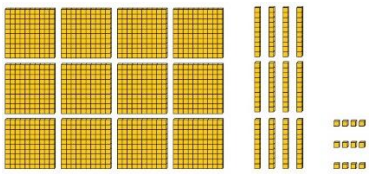
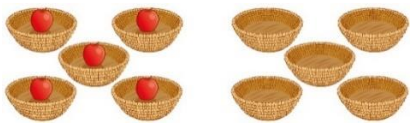
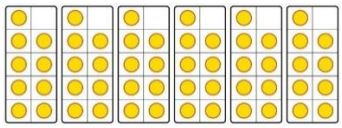
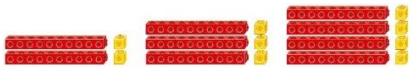
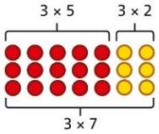
There are two groups of 14 and 1 remainder.	 *29 ÷ 2 = 14 remainder 1*	*67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*

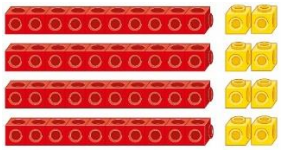
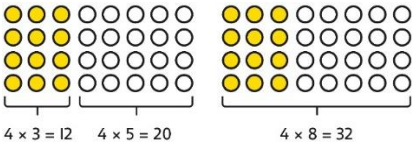
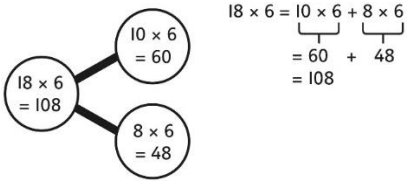
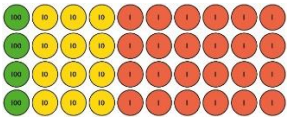
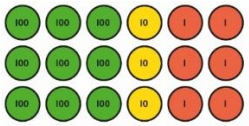
Year 4

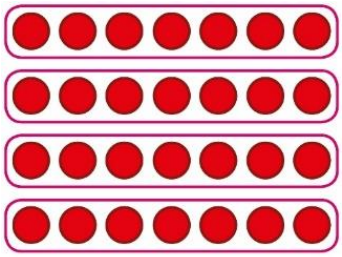
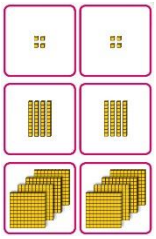
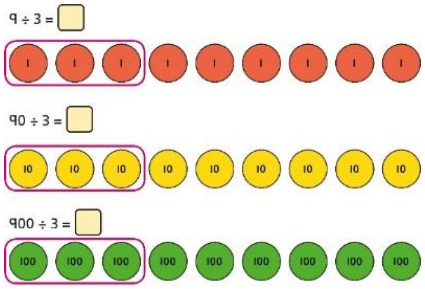
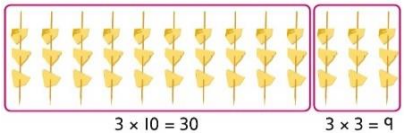
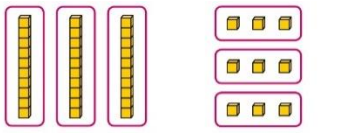
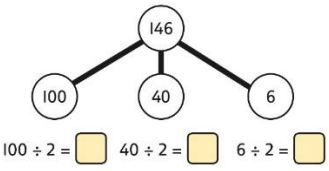
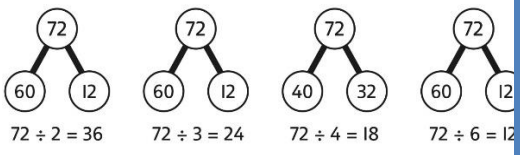
Concrete	Pictorial	Abstract
Use place value equipment to understand the place value of 4-digit numbers.  *4 thousands equal 4,000.* *1 thousand is 10 hundreds.*	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s.  *2,000 + 500 + 40 + 2 = 2,542*	Understand partitioning of 4-digit numbers, including numbers with digits of 0.  *5,000 + 60 + 8 = 5,068* Understand and read 4-digit numbers on a number line. 
Use unitising and known facts to support mental calculations. *Make 1,405 from place value equipment.* *Add 2,000.* *Now add the 1,000s.* *1 thousand + 2 thousands = 3 thousands* *1,405 + 2,000 = 3,405*	Use unitising and known facts to support mental calculations.  *I can add the 100s mentally.* *200 + 300 = 500* *So, 4,256 + 300 = 4,556*	Use unitising and known facts to support mental calculations. *4,256 + 300 = ?* *2 + 3 = 5* *200 + 300 = 500* *4,256 + 300 = 4,556*
Use place value equipment on a place value grid to organise thinking. Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers. *Use equipment.to show 1,905 + 775.* 	Use place value equipment to model required exchanges.	Use a column method to add, including exchanges.

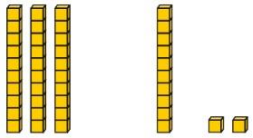
Why have only three columns been used for the second row? Why is the Thousands box empty? *Which columns will total 10 or more?*	 Include examples that exchange in more than one column.	 Include examples that exchange in more than one column.
	Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.  *I chose to work out 574 + 800, then subtract 1.*  *This is equivalent to 3,000 + 3,000.*	Use rounding and estimating on a number line to check the reasonableness of an addition.  *912 + 6,149 = ?* *I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.*
Use place value equipment to justify mental methods.	Use place value grids to support mental methods where appropriate.	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 – 2,000*

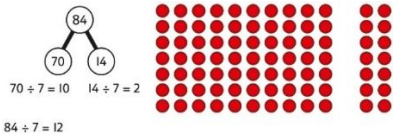
 *What number will be left if we take away 300?*	 *7,646 – 40 = 7,606*	*3 thousands – 2 thousands = 1 thousand* *3,501 – 2,000 = 1,501*
Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary. 	Represent place value equipment on a place value grid to subtract, including exchanges where needed. 	Use column subtraction, with understanding of the place value of any exchange required. 
Understand why two exchanges may be necessary. *2,502 – 243 = ?*  *I need to exchange a 10 for some 1s, but there are not any 10s here.* 	Make exchanges across more than one column where there is a zero as a place holder. *2,502 – 243 = ?* 	Make exchanges across more than one column where there is a zero as a place holder. *2,502 – 243 = ?* 

	Use bar models to represent subtractions where a part needs to be calculated.  *I can work out the total number of Yes votes using 5,762 – 2,899.* Bar models can also represent 'find the difference' as a subtraction problem.  	Use inverse operations to check subtractions. *I calculated 1,225 – 799 = 574.* *I will check by adding the parts.*   *The parts do not add to make 1,225.* *I must have made a mistake.*
Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds is 12 hundreds.*	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	Use known facts and understanding of place value and commutativity to multiply mentally. *4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understand the special cases of multiplying by 1 and 0.  *5 × 1 = 5* *5 × 0 = 0*	Represent the relationship between the ×9 table and the ×10 table.  Represent the ×11 table and ×12 tables in relation to the ×10 table.  *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8*	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table *I know that 7 × 5 = 35* *so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table *3 × 7 = 3 × 5 + 3 × 2*  ×9 table and ×10 table *6 × 10 = 60* *6 × 9 = 60 – 6*
Make multiplications by partitioning.	Understand how multiplication and partitioning are related through addition.	Use partitioning to multiply 2-digit numbers by a single digit.

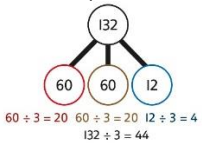
4 × 12 is 4 groups of 10 and 4 groups of 2.  *4 × 12 = 40 + 8*	 *4 × 3 = 12* *4 × 5 = 20* *12 + 20 = 32* *4 × 8 = 32*	*18 × 6 = ?*  *18 × 6 = 10 × 6 + 8 × 6* *= 60 + 48* *= 108*
Use place value equipment to make multiplications. *Make 4 × 136 using equipment.*  *I can work out how many 1s, 10s and 100s.* *There are 4 × 6 ones… 24 ones* *There are 4 × 3 tens … 12 tens* *There are 4 × 1 hundreds … 4 hundreds* *24 + 120 + 400 = 544*	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  3 1 2 × 3 9 3 6	Use the formal column method for up to 3-digit numbers multiplied by a single digit. 3 1 2 × 3 9 3 6 Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. 2 3 × 5 1 5 1 0 0 1 1 5 2 3 × 5 1 1 5 1
Represent situations by multiplying three numbers together.  *Each sheet has 2 × 5 stickers.* *There are 3 sheets.* *There are 5 × 2 × 3 stickers in total.* 5 × 2 × 3 = 30 10 × 3 = 30	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* *60 × 2 = 120*	Use knowledge of factors to simplify some multiplications. *24 × 5 = 12 × 2 × 5* 12 × 2 × 5 = 12 × 10 = 120 So, 24 × 5 = 120
Use objects to explore families of multiplication and division facts.	Represent divisions using an array.	Understand families of related multiplication and division facts. *I know that 5 × 7 = 35*

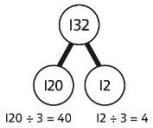
 *4 × 6 = 24* *24 is 6 groups of 4.* *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*	 28 ÷ 7 = 4	*so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* *5 = 35 ÷ 7*
Use place value equipment to understand how to use unitising to divide.  *8 ones divided into 2 equal groups* *4 ones in each group* *8 tens divided into 2 equal groups* *4 tens in each group* *8 hundreds divided into 2 equal groups* *4 hundreds in each group*	Represent divisions using place value equipment.  *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.*	Use known facts to divide 10s and 100s by a single digit. *15 ÷ 3 = 5* *150 ÷ 3 = 50* *1500 ÷ 3 = 500*
Partition into 10s and 1s to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* *142 ÷ 2 = 73*
Use place value equipment to explore why different partitions are needed. *42 ÷ 3 = ?* *I will split it into 30 and 12, so that I can divide by 3 more easily.*	Represent how to partition flexibly where needed. *84 ÷ 7 = ?* *I will partition into 70 and 14 because I am dividing by 7.*	Make decisions about appropriate partitioning based on the division required. 

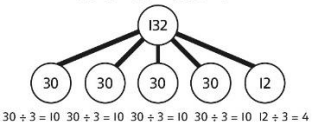




Understand that different partitions can be used to complete the same division.



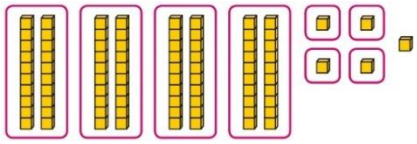




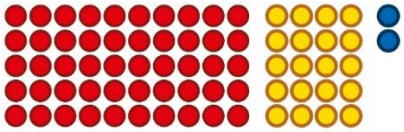
Use place value equipment to find remainders.

85 shared into 4 equal groups

There are 24, and 1 that cannot be shared.

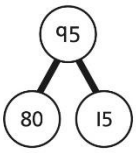


Represent the remainder as the part that cannot be shared equally.



72 ÷ 5 = 14 remainder 2

Understand how partitioning can reveal remainders of divisions.



80 ÷ 4 = 20
12 ÷ 4 = 3

95 ÷ 4 = 23 remainder 3

Upper Key stage 2

In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number

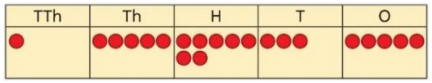
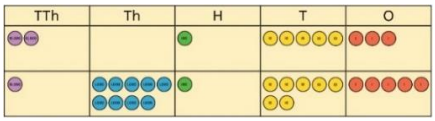
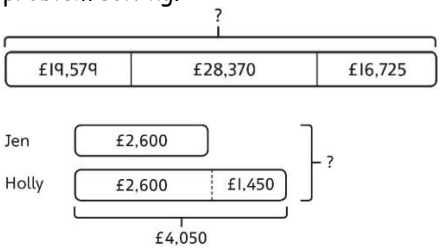
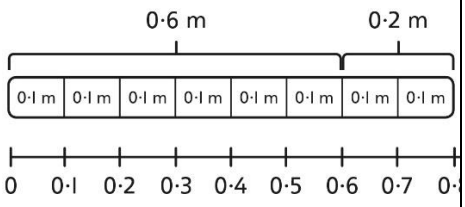
Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

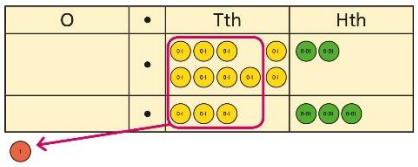
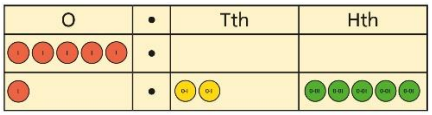
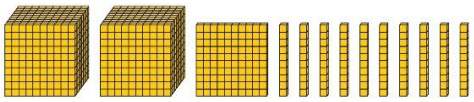
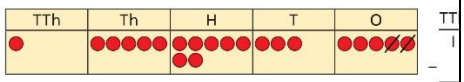
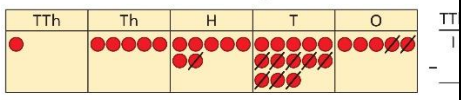
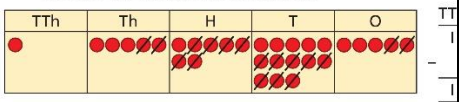
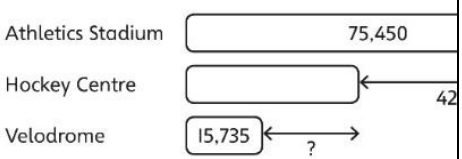
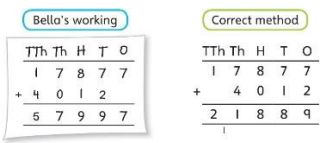
Multiplication and division: Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6.

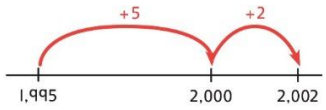
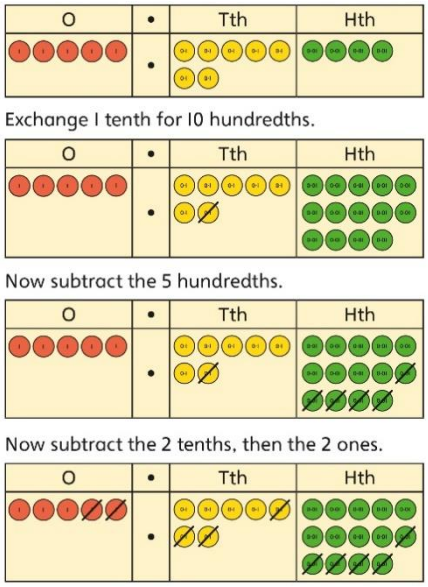
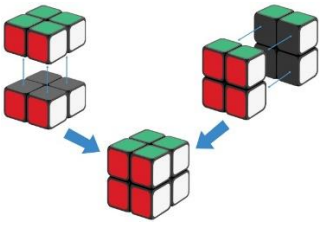
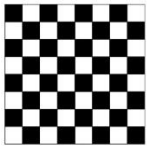
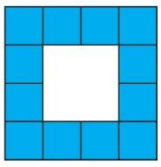
Fractions: Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

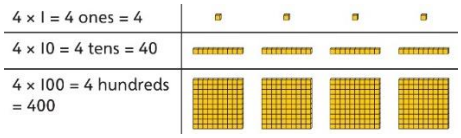
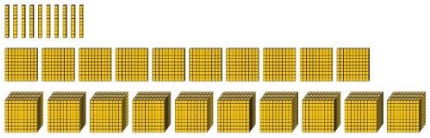
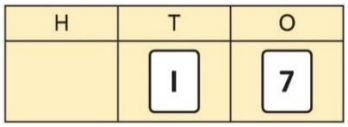
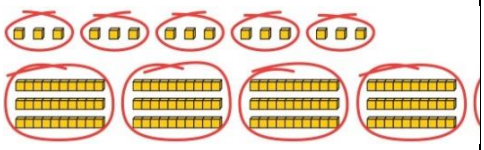
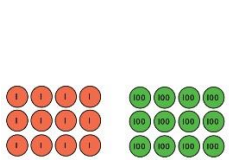
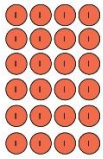
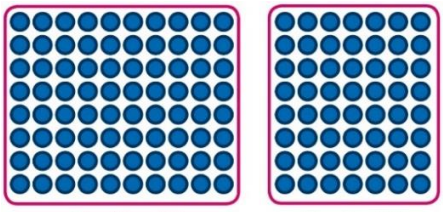
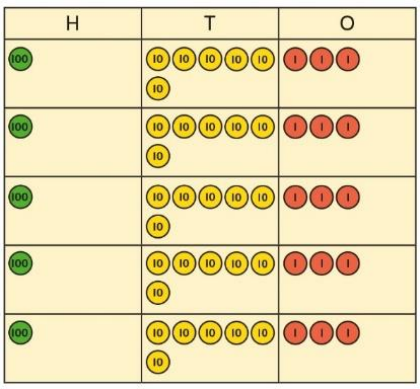
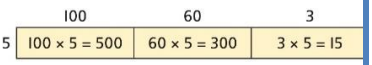
The tables below set out the expected models and images, and informal and formal methods of calculation for teachers to use, model and demonstrate to children at this stage of learnin

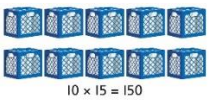
Year 5

	Concrete	Pictorial	Abstract
Year 5 Addition			
Column addition with whole numbers	Use place value equipment to represent additions. *Add a row of counters onto the place value grid to show 15,735 + 4,012.* 	Represent additions, using place value equipment on a place value grid alongside written methods.  *I need to exchange 10 tens for a 100.* TTh Th H T O 2 0 1 5 3 + 1 9 1 7 5 3 9 3 2 8	Use column addition, including exchanges. TTh Th H T O 1 9 1 7 5 + 1 8 4 1 7 3 7 5 9 2
Representing additions		Bar models represent addition of two or more numbers in the context of problem solving.  Th H T O 2 6 0 0 + 1 4 5 0 4 0 5 0 Th H T O 2 6 0 0 + 4 0 5 0 6 6 5 0	Use approximation to check whether answers are reasonable. TTh Th H T O 2 3 4 0 5 + 7 8 9 2 2 0 2 9 7 TTh Th H T O 2 3 4 0 5 + 7 8 9 2 3 1 2 9 7 *I will use 23,000 + 8,000 to check.*
Adding tenths	Link measure with addition of decimals. *Two lengths of fencing are 0·6 m and 0·2 m.* *How long are they when added together?* 	Use a bar model with a number line to add tenths.  *0·6 + 0·2 = 0·8* *6 tenths + 2 tenths = 8 tenths*	Understand the link with adding fractions. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ *6 tenths + 2 tenths = 8 tenths* *0·6 + 0·2 = 0·8*
Adding decimals using column addition	Use place value equipment to represent additions. *Show 0·23 + 0·45 using place value counters.*	Use place value equipment on a place value grid to represent additions. Represent exchange where necessary.	Add using a column method, ensuring that children understand the link with place value. O · Tth Hth 0 · 2 3 + 0 · 4 5 0 · 6 8

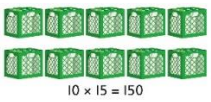
		 Include examples where the numbers of decimal places are different. 	Include exchange where required, alongside an understanding of place value. O · Tth Hth 0 · 9 2 + 0 · 3 3 1 · 2 5 Include additions where the numbers of decimal places are different. *3.4 + 0.65 = ?* O · Tth Hth 3 · 4 0 + 0 · 6 5
Year 5 Subtraction			
Column subtraction with whole numbers	Use place value equipment to understand where exchanges are required. *2,250 – 1,070* 	Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. *15,735 – 2,582 = 13,153*  Now subtract the 10s. Exchange 1 hundred for 10 tens.  Subtract the 100s, 1,000s and 10,000s. 	Use column subtraction methods with exchange where required. TTh Th H T O 5 11 1 6 2 0 9 7 – 1 8 5 3 4 4 3 5 6 3 *62,097 – 18,534 = 43,563*
Checking strategies and representing subtractions		Bar models represent subtractions in problem contexts, including 'find the difference'. 	Children can explain the mistake made when the columns have not been ordered correctly.  Use approximation to check calculations. *I calculated 18,000 + 4,000 mentally to check my subtraction.*
Choosing efficient methods			To subtract two large numbers that are close, children find the difference by counting on.

			2,002 – 1,995 = ?  Use addition to check subtractions. *I calculated 7,546 – 2,355 = 5,191.* *I will check using the inverse.*
Subtracting decimals	Explore complements to a whole number by working in the context of length.  1 m – ☐ m = ☐ m *1 – 0·49 = ?*	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 – 2·25 = ?* 	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 – 3·75 = ?* O · Tth Hth Thth 3 · 9 2 1 – 3 · 7 5 0 ·
Year 5 Multiplication			
Understanding factors	Use cubes or counters to explore the meaning of 'square numbers'. *25 is a square number because it is made from 5 rows of 5.* Use cubes to explore cube numbers.  *8 is a cube number.*	Use images to explore examples and non-examples of square numbers.  *8 × 8 = 64* $8^2 = 64$ 	Understand the pattern of square numbers in the multiplication tables. Use a multiplication grid to circle each square number. Can children spot a pattern?

		12 is not a square number, because you cannot multiply a whole number by itself to make 12.	
Multiplying by 10, 100 and 1,000	Use place value equipment to multiply by 10, 100 and 1,000 by unitising.  	Understand the effect of repeated multiplication by 10. 	Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.  *17 × 10 = 170* *17 × 100 = 17 × 10 × 10 = 1,700* *17 × 1,000 = 17 × 10 × 10 × 10 = 17,000*
Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising.  *5 groups of 3 ones is 15 ones.* *5 groups of 3 tens is 15 tens.* *So, I know that 5 groups of 3 thousands would be 15 thousands.*	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.  *4 × 3 = 12* *4 × 300 = 1,200* *2,400*  *6 × 4 = 24* *6 × 400 =*	Use known facts and unitising to multiply. *5 × 4 = 20* *5 × 40 = 200* *5 × 400 = 2,000* *5 × 4,000 – 20,000* *5,000 × 4 = 20,000*
Multiplying up to 4-digit numbers by a single digit	Explore how to use partitioning to multiply efficiently. *8 × 17 = ?*  8 × 10 = 80 8 × 7 = 56 80 + 56 = 136 *So, 8 × 17 = 136*	Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.  	Use an area model and then add the parts.  Use a column multiplication, including any required exchanges. 1 3 6 × 6 8 1 6 2 3
Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?*	Use an area model and add the parts. *28 × 15 = ?*	Use column multiplication, ensuring understanding of place value at each stage.



10 × 15 = 150



10 × 15 = 150

3 × 15 = 45

There are 345 bottles of milk in total.

H	T	O
1	5	0
1	5	0
+	4	5
3	4	5
	1	

23 × 15 = 345

	20 m	8 m
10 m	20 × 10 = 200 m²	8 × 10 = 80 m²
5 m	20 × 5 = 100 m²	8 × 5 = 40 m²

H	T	O
2	0	0
1	0	0
	8	0
+	4	0
4	2	0
	1	

28 × 15 = 420

$$\begin{array}{rr} & 34 \\ \times & 27 \\ \hline & 238 \\ & {\scriptstyle 2} \\ \hline \\ \hline & 34 \\ \times & 27 \\ \hline & 238 \\ & {\scriptstyle 2} \\ & 680 \\ \hline \\ \hline & 34 \\ \times & 27 \\ \hline & 238 \\ & {\scriptstyle 2} \\ & 680 \\ & 918 \\ & {\scriptstyle 1} \end{array} \quad \begin{array}{l} 34 \times 7 \\ \\ \\ \\ \\ 34 \times 7 \\ 34 \times 20 \\ \\ \\ 34 \times 7 \\ 34 \times 20 \\ 34 \times 27 \end{array}$$

Multiplying up to 4-digits by 2-digits

Use the area model then add the parts.

	100	40	3
10			
2			

143 × 12 = 1,716

There are 1,716 boxes of cereal in total.

143 × 12 = 1,716

Use column multiplication, ensuring understanding of place value at each stage.

$$\begin{array}{rrrr} & 1 & 4 & 3 \\ \times & & 1 & 2 \\ \hline & 2 & 8 & 6 \\ 1 & 4 & 3 & 0 \\ \hline 1 & 7 & 1 & 6 \\ & 1 & & \end{array} \quad \begin{array}{l} 143 \times 2 \\ 143 \times 10 \\ 143 \times 12 \end{array}$$

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

1,274 × 32 = ?
First multiply 1,274 by 2.

$$\begin{array}{rrrrr} & 1 & 2 & 7 & 4 \\ \times & & & 3 & 2 \\ \hline & 2 & 5 & {\scriptstyle 1}4 & 8 \\ \hline \\ \hline \end{array} \quad 1{,}274 \times 2$$

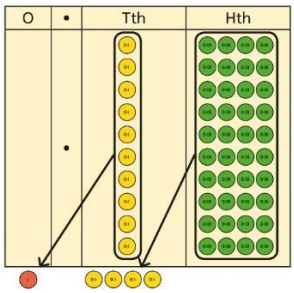
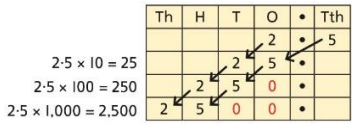
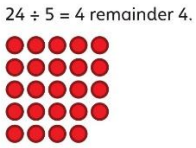
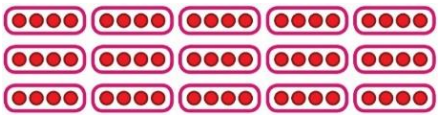
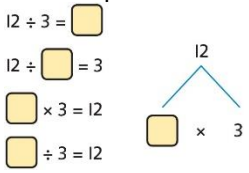
Then multiply 1,274 by 30.

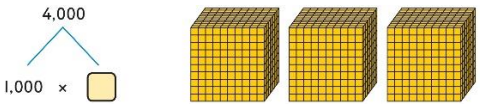
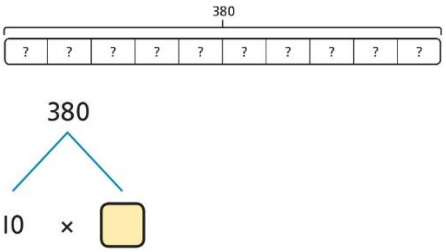
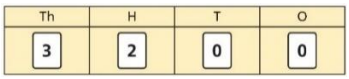
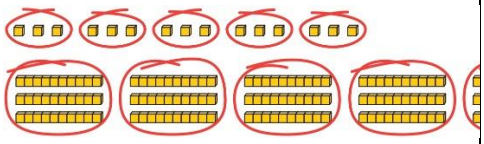
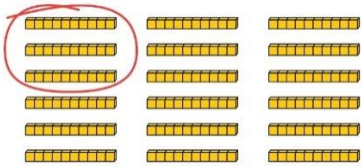
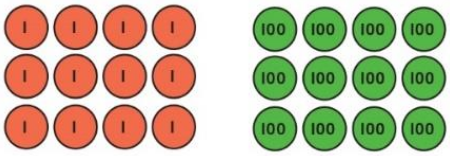
$$\begin{array}{rrrrr} & 1 & 2 & 7 & 4 \\ \times & & & 3 & 2 \\ \hline & 2 & 5 & {\scriptstyle 1}4 & 8 \\ 3 & 8 & {\scriptstyle 2}2 & {\scriptstyle 1}2 & 0 \\ \hline \\ \hline \end{array} \quad \begin{array}{l} 1{,}274 \times 2 \\ 1{,}274 \times 30 \end{array}$$

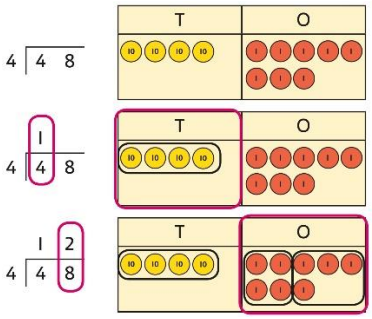
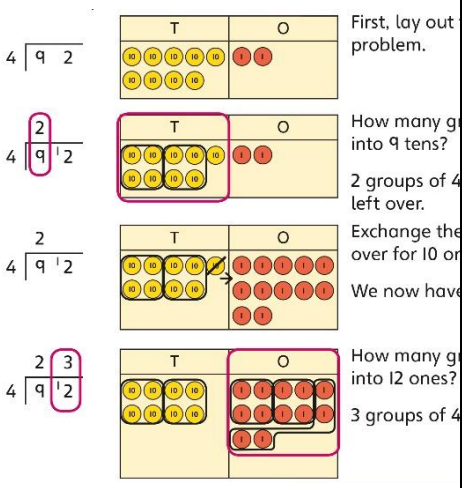
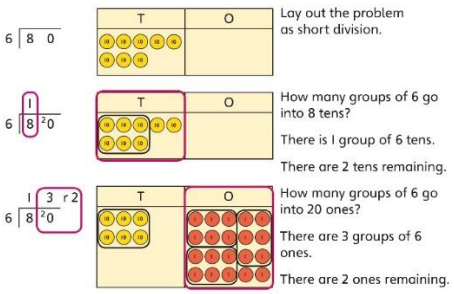
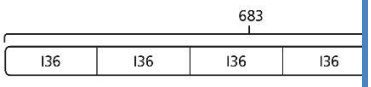
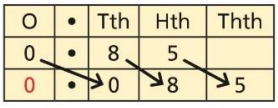
Finally, find the total.

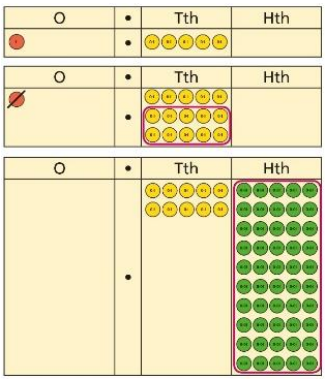
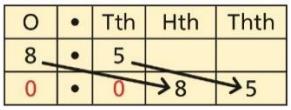
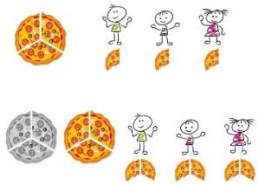
$$\begin{array}{rrrrr} & 1 & 2 & 7 & 4 \\ \times & & & 3 & 2 \\ \hline & 2 & 5 & {\scriptstyle 1}4 & 8 \\ 3 & 8 & {\scriptstyle 2}2 & {\scriptstyle 1}2 & 0 \\ \hline 4 & 0 & 7 & 6 & 8 \\ & 1 & & & \end{array} \quad \begin{array}{l} 1{,}274 \times 2 \\ 1{,}274 \times 30 \\ 1{,}274 \times 32 \end{array}$$

1,274 × 32 = 40,768

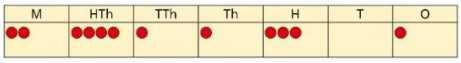
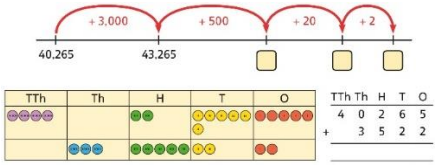
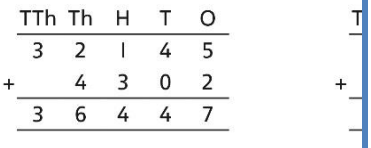
Multiplying decimals by 10, 100 and 1,000	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid.  *0·14 × 10 = 1·4*	Understand how this exchange is represented on a place value chart. 
Year 5 Division			
Understanding factors and prime numbers	Use equipment to explore the factors of a given number.  *24 ÷ 3 = 8* *24 ÷ 8 = 3* *8 and 3 are factors of 24 because they divide 24 exactly.*  *5 is not a factor of 24 because there is a remainder.*	Understand that prime numbers are numbers with exactly two factors. *13 ÷ 1 = 13* *13 ÷ 2 = 6 r 1* *13 ÷ 4 = 4 r 1* *1 and 13 are the only factors of 13.* *13 is a prime number.*	Understand how to recognise prime and composite numbers. *I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.* *I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.* *I know that 1 is not a prime number, as it has only 1 factor.*
Understanding inverse operations and the link with multiplication, grouping and sharing	Use equipment to group and share and to explore the calculations that are present. *I have 28 counters.* *I made 7 groups of 4. There are 28 in total.* *I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.* *I have 28 in total. I made groups of 4. There are 7 equal groups.*	Represent multiplicative relationships and explore the families of division facts.  *60 ÷ 4 = 15* *60 ÷ 15 = 4*	Represent the different multiplicative relationships to solve problems requiring inverse operations.  Understand missing number problems for division calculations and know how to solve them using inverse operations. *22 ÷ ? = 2* *22 ÷ 2 = ?* *? ÷ 2 = 22* *? ÷ 22 = 2*
Dividing whole numbers by 10, 100 and 1,000	Use place value equipment to support unitising for division. *4,000 ÷ 1,000*	Use a bar model to support dividing by unitising. *380 ÷ 10 = 38*	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.

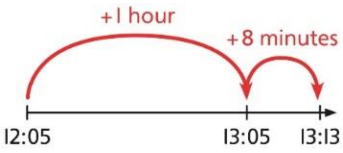
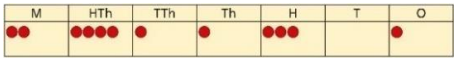
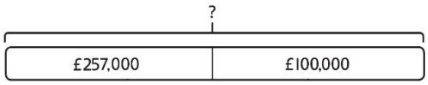
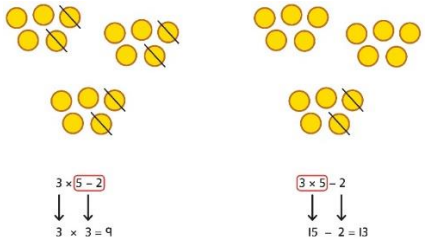
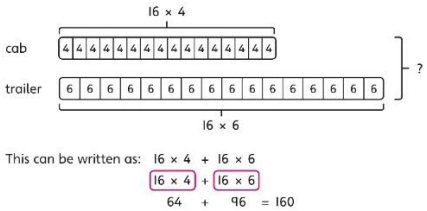
	 *4,000 is 4 thousands.* *4 × 1,000= 4,000* *So, 4,000 ÷ 1,000 = 4*	 *380 is 38 tens.* *38 × 10 = 380* *10 × 38 = 380* *So, 380 ÷ 10 = 38*	 *3,200 ÷ 100 = ?* *3,200 is 3 thousands and 2 hundreds.* *200 ÷ 100 = 2* *3,000 ÷ 100 = 30* *3,200 ÷ 100 = 32* *So, the digits will move two places to the right.*
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising.  *15 ones put into groups of 3 ones. There are 5 groups.* *15 ÷ 3 = 5* *15 tens put into groups of 3 tens. There are 5 groups.* *150 ÷ 30 = 5*	Represent related facts with place value equipment when dividing by unitising.  *180 is 18 tens.* *18 tens divided into groups of 3 tens. There are 6 groups.* *180 ÷ 30 = 6*  *12 ones divided into groups of 4. There are 3 groups.* *12 hundreds divided into groups of 4 hundreds. There are 3 groups.* *1200 ÷ 400 = 3*	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. *3,000 ÷ 5 = 600* *3,000 ÷ 50 = 60* *3,000 ÷ 500 = 6* *5 × 600 = 3,000* *50 × 60 = 3,000* *500 × 6 = 3,000*
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. *268 ÷ 2 = ?* *There is 1 group of 2 hundreds.* *There are 3 groups of 2 tens.* *There are 4 groups of 2 ones.* *264 ÷ 2 = 134*	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.	Use short division for up to 4-digit numbers divided by a single digit. $$\begin{array}{r} 0\ 5\ 5\ 6 \\ 7\overline{)\,3\ {}^{3}8\ {}^{3}9\ {}^{4}2} \end{array}$$ *3,892 ÷ 7 = 556* Use multiplication to check.

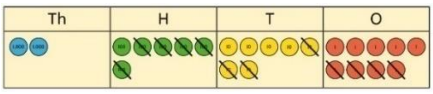
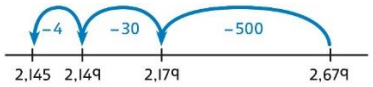
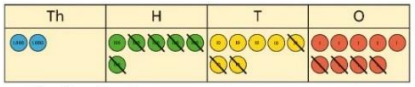
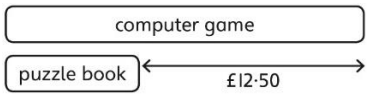
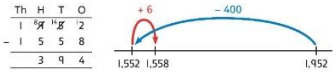
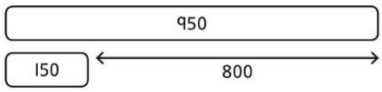
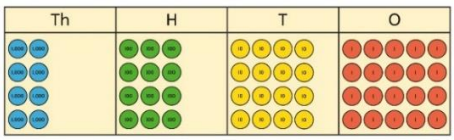
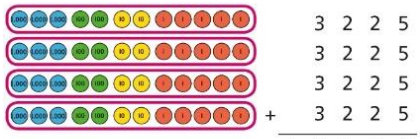
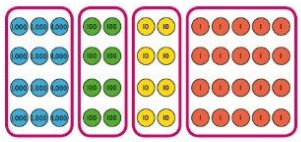
		 Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* Work with divisions that require exchange. 	*556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*
Understanding remainders	Understand remainders using concrete versions of a problem. *80 cakes divided into trays of 6.*  *80 cakes in total. They make 13 groups of 6, with 2 remaining.*	Use short division and understand remainders as the last remaining 1s. 	In problem solving contexts, represent divisions including remainders with a bar model.  *683 = 136 × 5 + 3* *683 ÷ 5 = 136 r 3*
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. *2 ones are 20 tenths.* *20 tenths divided by 10 is 2 tenths.*	Represent division using exchange on a place value grid.	Understand the movement of digits on a place value grid.  *0·85 ÷ 10 = 0·085*

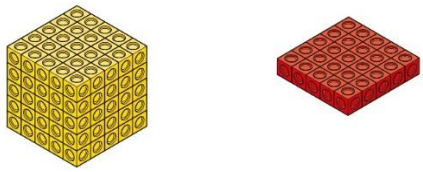
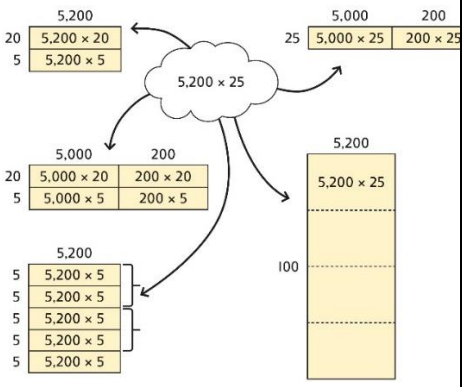
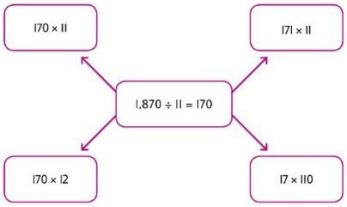
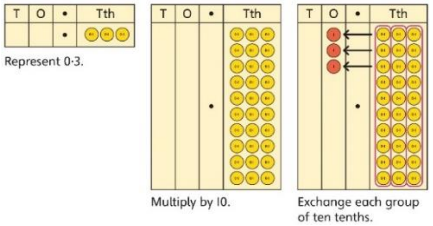
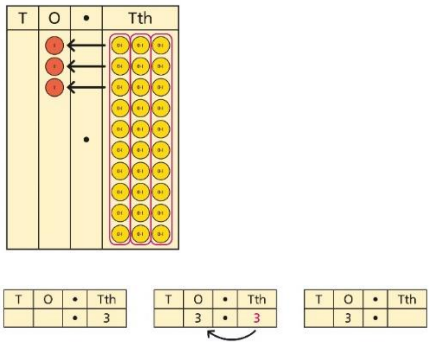
		 *1·5 is 1 one and 5 tenths.* *This is equivalent to 10 tenths and 50 hundredths.* *10 tenths divided by 10 is 1 tenth.* *50 hundredths divided by 10 is 5 hundredths.* *1·5 divided by 10 is 1 tenth and 5 hundredths.* *1·5 ÷ 10 = 0.15*	 *8·5 ÷ 100 = 0·085*
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. *1 whole shared between 3 people.* *Each person receives one-third.* 	Use a bar model and other fraction representations to show the link between fractions and division.  $1 \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$

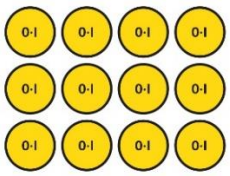
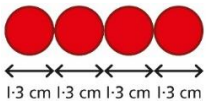
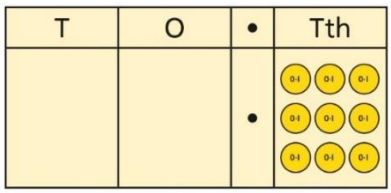
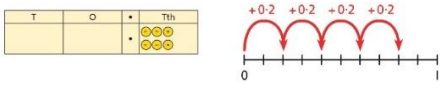
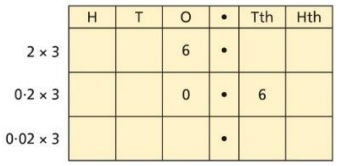
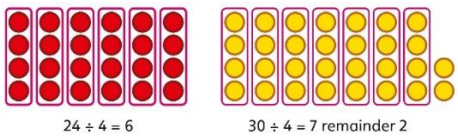
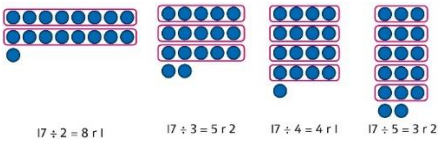
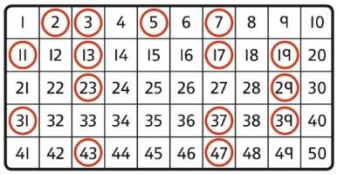
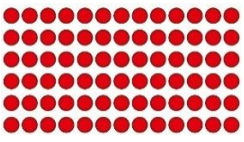
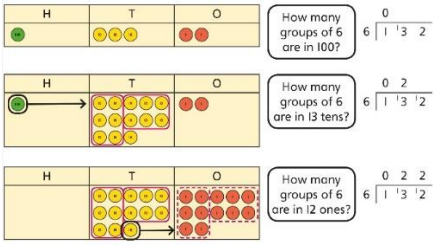
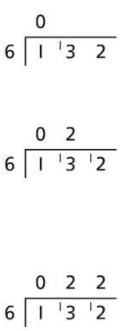
Year 6

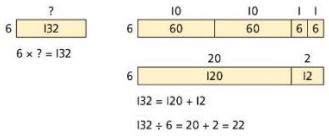
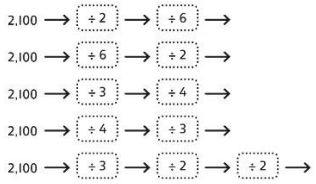
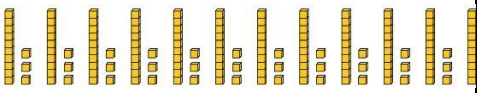
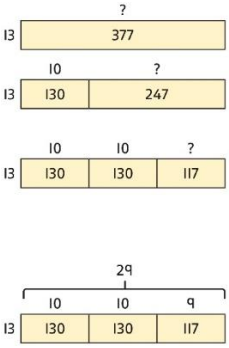
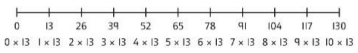
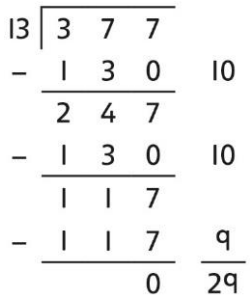
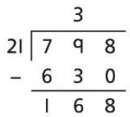
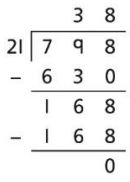
	Concrete	**Pictorial**	**Abstract**
Year 6 Addition			
Comparing and selecting efficient methods	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods. 	Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.  Use bar model and number line representations to model addition in problem-solving and measure contexts.	Use column addition where mental methods are not efficient. Recognise common errors with column addition. *32,145 + 4,302 = ?*  *Which method has been completed accurately?* *What mistake has been made?*

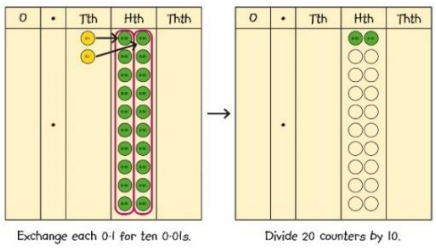
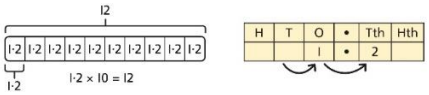
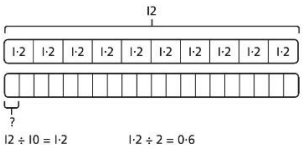
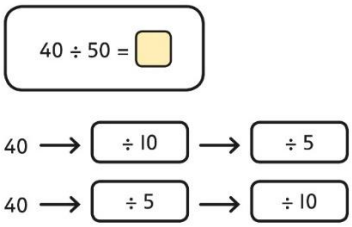
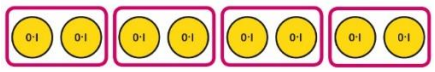
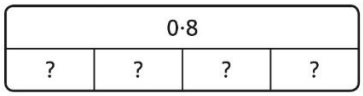
			Column methods are also used for decimal additions where mental methods are not efficient. H T O · Tth Hth 1 4 0 · 0 9 + 4 9 · 8 9 1 8 9 · 9 8 1
Selecting mental methods for larger numbers where appropriate	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.  *2,411,301 + 500,000 = ?* *This would be 5 more counters in the HTh place.* *So, the total is 2,911,301.* *2,411,301 + 500,000 = 2,911,301*	Use a bar model to support thinking in addition problems. *257,000 + 99,000 = ?*  *I added 100 thousands then subtracted* *1 thousand.* *257 thousands + 100 thousands = 357 thousands* *257,000 + 100,000 = 357,000* *357,000 – 1,000 = 356,000* *So, 257,000 + 99,000 = 356,000*	Use place value and unitising to support mental calculations with larger numbers. *195,000 + 6,000 = ?* *195 + 5 + 1 = 201* *195 thousands + 6 thousands = 201 thousands* *So, 195,000 + 6,000 = 201,000*
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 × 5 – 2 = ?* 	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. 	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. *4 + 6 × 16* *4 + 96 = 100* *(4 + 6) × 16* *10 × 16 = 160*
Year 6 Subtraction			
Comparing and selecting efficient methods	Use counters on a place value grid to represent subtractions of larger numbers.	Compare subtraction methods alongside place value representations.	Compare and select methods. Use column subtraction when mental methods are not efficient.

		  Th H T O 2 6 7 9 − 5 3 4 2 1 4 5 Use a bar model to represent calculations, including 'find the difference' with two bars as comparison. 	Use two different methods for one calculation as a checking strategy.  Use column subtraction for decimal problems, including in the context of measure. H T O · Tth Hth 3 0 9 · 6 0 − 2 0 6 · 4 0 1 0 3 · 2 0
Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. *950,000 – 150,000* *That is 950 thousands – 150 thousands*  *So, the difference is 800 thousands.* *950,000 – 150,000 = 800,000*	Subtract efficiently from powers of 10. *10,000 – 500 = ?*
Year 6 Multiplication			
Multiplying up to a 4-digit number by a single digit number	Use equipment to explore multiplications.  *4 groups of 2,345* *This is a multiplication:* *4 × 2,345* *2,345 × 4*	Use place value equipment to compare methods. **Method 1**  3 2 2 5 3 2 2 5 3 2 2 5 + 3 2 2 5 1 2 9 0 0 1 1 2 **Method 2**  4 × 3,000 4 × 200 4 × 20 4 × 5 12,000 + 800 + 80 + 20 = 12,900	Understand area model and short multiplication. Compare and select appropriate methods for specific multiplications. **Method 3** 3,000 200 20 5 4 | 12,000 | 800 | 80 | 20 12,000 + 800 + 80 + 20 = 12,900 **Method 4** 3 2 2 5 × 4 1 2 9 0 0 1 2
Multiplying up to a 4-digit number by a 2-digit number		Use an area model alongside written multiplication.	Use compact column multiplication with understanding of place value at all stages.

		Method I 1,000 200 30 5 20 | 20,000 | 4,000 | 600 | 100 1 | 1,000 | 200 | 30 | 5 1 2 3 5 × 2 1 5 — 1 × 5 3 0 — 1 × 30 2 0 0 — 1 × 200 1 0 0 0 — 1 × 1,000 1 0 0 — 20 × 5 6 0 0 — 20 × 30 4 0 0 0 — 20 × 200 2 0 0 0 0 — 20 × 1,000 2 5 9 3 5 — 21 × 1,235	1 2 3 5 × 2 1 1 2 3 5 — 1 × 1,235 2 4 7 0 0 — 20 × 1,235 2 5 9 3 5 — 21 × 1,235
Using knowledge of factors and partitions to compare methods for multiplications	Use equipment to understand square numbers and cube numbers.  $5 \times 5 = 5^2 = 25$ $5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately.  Represent and compare methods using a bar model.	Use a known fact to generate families of related facts.  Use factors to calculate efficiently. 15×16 $= 3 \times 5 \times 2 \times 8$ $= 3 \times 8 \times 2 \times 5$ $= 24 \times 10$ $= 240$
Multiplying by 10, 100 and 1,000	Use place value equipment to explore exchange in decimal multiplication.  $0{\cdot}3 \times 10 = ?$ $0{\cdot}3$ is 3 tenths. 10×3 tenths are 30 tenths. 30 tenths are equivalent to 3 ones.	Understand how the exchange affects decimal numbers on a place value grid.  $0{\cdot}3 \times 10 = 3$	Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000. $8 \times 100 = 800$ $8 \times 300 = 800 \times 3$ $= 2{,}400$ $2{\cdot}5 \times 10 = 25$ $2{\cdot}5 \times 20 = 2{\cdot}5 \times 10 \times 2$ $= 50$
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.	Represent calculations on a place value grid.	Use known facts to multiply decimals. $4 \times 3 = 12$ $4 \times 0{\cdot}3 = 1{\cdot}2$ $4 \times 0{\cdot}03 = 0{\cdot}12$

	 *3 groups of 4 tenths is 12 tenths.* *4 groups of 3 tenths is 12 tenths.*  *4 × 1 cm = 4 cm* *4 × 0·3 cm = 1.2 cm* *4 × 1·3 = 4 + 1·2 = 5·2 cm*	$3 \times 3 = 9$ $3 \times 0{\cdot}3 = 0{\cdot}9$  Understand the link between multiplying decimals and repeated addition. 	*20 × 5 = 100* *20 × 0·5 = 10* *20 × 0·05 = 1* Find families of facts from a known multiplication. *I know that 18 × 4 = 72.* *This can help me work out:* *1·8 × 4 = ?* *18 × 0·4 = ?* *180 × 0·4 = ?* *18 × 0·04 = ?* Use a place value grid to understand the effects of multiplying decimals. 
Year 6 Division			
Understanding factors	Use equipment to explore different factors of a number.  *4 is a factor of 24 but is not a factor of 30.*	Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders. 	Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number. 
Dividing by a single digit	Use equipment to make groups from a total.  *There are 78 in total.* *There are 6 groups of 13.* *There are 13 groups of 6.*		Use short division to divide by a single digit. 

			Use an area model to link multiplication and division. 
Dividing by a 2-digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. *1,260 ÷ 14 = ?*  *1,260 ÷ 2 = 630* *630 ÷ 7 = 90* *1,260 ÷ 14 = 90*	Use factors and repeated division where appropriate. *2,100 ÷ 12 = ?* 
Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.  *182 divided into groups of 13.* *There are 14 groups.*	Use an area model alongside written division to model the process. *377 ÷ 13 = ?*  *377 ÷ 13 = 29*	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. *377 ÷ 13 = ?*   *377 ÷ 13 = 29* A slightly different layout may be used, with the division completed above rather than at the side.  

			Divisions with a remainder explored in problem-solving contexts.
Dividing by 10, 100 and 1,000	Use place value equipment to explore division as exchange.  Exchange each 0·1 for ten 0·01s. Divide 20 counters by 10. *0·2 is 2 tenths.* *2 tenths is equivalent to 20 hundredths.* *20 hundredths divided by 10 is 2 hundredths.*	Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.  Understand how to divide using division by 10, 100 and 1,000. *12 ÷ 20 = ?*  	Use knowledge of factors to divide by multiples of 10, 100 and 1,000.  *40 ÷ 5 = 8* *8 ÷ 10 = 0·8* *So, 40 ÷ 50 = 0·8*
Dividing decimals	Use place value equipment to explore division of decimals.  *8 tenths divided into 4 groups. 2 tenths in each group.*	Use a bar model to represent divisions.  4 × 2 = 8 8 ÷ 4 = 2 So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2	Use short division to divide decimals with up to 2 decimal places.